The Power of Tabulation Hashing

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Joint work with Mihai Pătraşcu. Some of it found in Proc. STOC'11.

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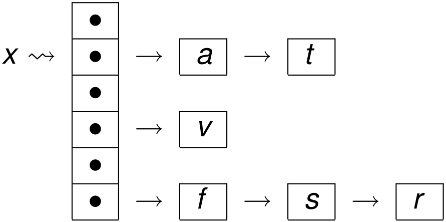
- Providing algorithmically important probabilisitic guarantees akin to those of truly random hashing, yet easy to implement.
- Bridging theory (assuming truly random hashing) with practice (needing something implementable).

Applications of Hashing $\rightsquigarrow$

Hash tables (n keys and $2n$ hashes: expect 1/2 keys per hash)

- chaining: follow pointers



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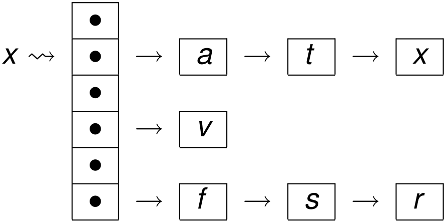

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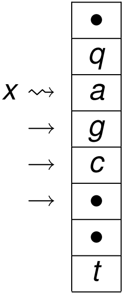

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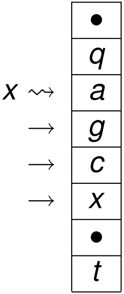

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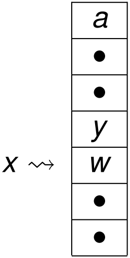

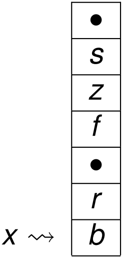

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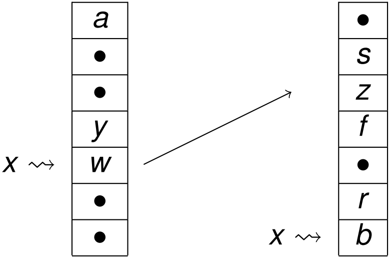

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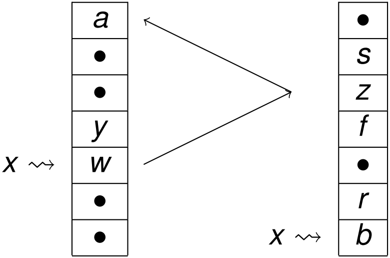

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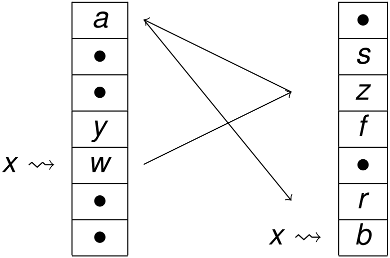

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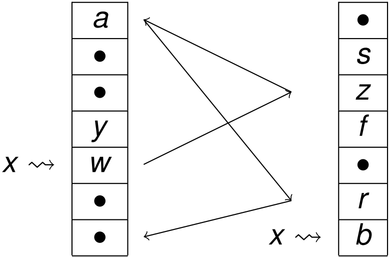

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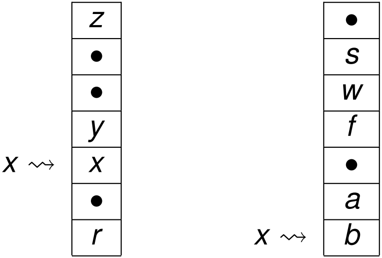

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Sketching, streaming, and sampling:

- second moment estimation: $F_2(\bar{x}) = \sum_i x_i^2$

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Sketching, streaming, and sampling:

- second moment estimation: $F_2(\bar{x}) = \sum_i x_i^2$
- sketch A and B to later find $|A \cap B|/|A \cup B|$

$$|A \cap B|/|A \cup B| = \Pr_h[\min h(A) = \min h(B)]$$

We need h to be ε-minwise independent:

$$(\forall)x \notin S : \qquad \Pr[h(x) < \min h(S)] = \frac{1 \pm \varepsilon}{|S| + 1}$$

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Important outside theory. These simple practical hash tables often bottlenecks in the processing of data—substantial fraction of worlds computational resources spent here.

Carter & Wegman (1977)

We do not have space for truly random hash functions, but

Family $\mathcal{H} = \{h : [u] \to [b]\}$ k-independent iff for random $h \in \mathcal{H}$:

- $(\forall) x \in [u]$, $h(x)$ is uniform in $[b]$;
- $(\forall) x_1, \ldots, x_k \in [u]$, $h(x_1), \ldots, h(x_k)$ are independent.

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Prototypical example: degree $k - 1$ polynomial

- $u = b$ prime;
- choose $a_0, a_1, \ldots, a_{k-1}$ randomly in $[u]$;
- $h(x) = \left(a_0 + a_1 x + \cdots + a_{k-1} x^{k-1}\right)$ mod u.

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Many solutions for k-independent hashing proposed, but generally slow for $k > 3$ and too slow for $k > 5$.

How much independence needed?

Chaining $\mathbf{E}[t] = O(1)$	2	
$\mathbf{E}[t^k] = O(1)$	$2k+1$	
$t = O\left(\frac{\lg n}{\lg\lg n}\right)$ w.h.p.	$\Theta\left(\frac{\lg n}{\lg\lg n}\right)$	
Linear probing	≤ 5 [Pagh^2, Ružić'07]	≥ 5 **[PT ICALP'10]**
Cuckoo hashing	$O(\lg n)$	≥ 6 [Cohen, Kane'05]
F_2 estimation	4 [Alon, Mathias, Szegedy'99]	
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Independence has been the ruling measure for quality of hash functions for 30$+$ years, but is it right?

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- Space $cN^{1/c}$ and time $O(c)$. With 8-bit characters, each R_i has 256 entries and fit in L1 cache.

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- Not 4-independent: $h(a_1 a_2) \oplus h(a_1 b_2) \oplus h(b_1 a_2) \oplus h(b_1 b_2)$

$$\begin{aligned} = \quad & (R_1[a_1] \oplus R_2[a_2]) \oplus (R_1[a_1] \oplus R_2[b_2]) \oplus \\ & (R_1[b_1] \oplus R_2[a_2]) \oplus (R_1[b_1] \oplus R_2[b_2]) \; = \; 0. \end{aligned}$$

How much independence needed? Wrong question

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New result: Despite its 4-dependence, simple tabulation suffices for all the above applications:

> *One simple and fast hashing scheme for almost all your needs.*

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Knuth recommends simple tabulation but cites only 3-independence as mathematical quality.
We prove that dependence of simple tabulation is not harmful in any of the above applications.

Chaining/hashing into bins

Theorem Consider hashing n balls into $m \geq n^{1-1/(2c)}$ bins by simple tabulation. Let q be an additional *query ball*, and define X_q as the number of regular balls that hash into a bin chosen as a function of $h(q)$. Let $\mu = \mathbf{E}[X_q] = \frac{n}{m}$. The following probability bounds hold for any constant γ:

$$\Pr[X_q \geq (1+\delta)\mu] \leq \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{\Omega(\mu)} + m^{-\gamma}$$

$$\Pr[X_q \leq (1-\delta)\mu] \leq \left(\frac{e^{-\delta}}{(1-\delta)^{(1-\delta)}}\right)^{\Omega(\mu)} + m^{-\gamma}$$

With $m \leq n$ bins, every bin gets

$$n/m \pm O\left(\sqrt{n/m}\log^c n\right).$$

keys with probability $1 - n^{-\gamma}$.

Hashing into many bins

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Nothing like this lemma holds if we instead of simple tabulation assumed k-independent hashing with $k = O(1)$.

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Proof that for any positive constants ε, γ, if we hash n keys into m bins and $n \leq m^{1-\varepsilon}$, then all bins get less than $d = 2^{(1+\gamma)/\varepsilon}$ keys with probability $\geq 1 - m^{-\gamma}$.

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- Return $\{x\} \cup U'$ where U' independent subset of T'.

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- There are $\binom{n}{u} < n^u$ sets U of u keys to consider.

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- There are $\binom{n}{u} < n^u$ sets U of u keys to consider.
- Each such U hash to *one* bin with probability $1/m^{u-1}$.

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Claim 2 The probability that there exists $u = (1+\gamma)/\varepsilon$ keys hashing independently to the same bin is $m^{-\gamma}$.

- There are $\binom{n}{u} < n^u$ sets U of u keys to consider.
- Each such U hash to *one* bin with probability $1/m^{u-1}$.
- Propability bound over all U is

$$n^u m^{u-1} \leq m^{(1-\varepsilon)u+1-u} = m^{1-\varepsilon u} = m^{-\gamma}.$$

Hashing into many bins

Lemma If we hash n keys into $n^{1+\Omega(1)}$ bins, then all bins get $O(1)$ keys w.h.p.

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▸ Short of time?

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- If the X_G were really independent, by Chernoff

$$\Pr[X \geq (1+\delta)\mu] \leq \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{\mu/d}$$

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- So claim false implies S in hypercube of size $< \left(n^{1/c}\right)^c = n$.

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- But always $\mathbf{E}[X_{G_{(i,a)}}] = |X_{G_{(i,a)}}|/m$. Moreover $X_{G_{(i,a)}} \leq d$.
- Good enough for Chernoff bounds.

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W.h.p., the contribution X to given obeys Chernoff

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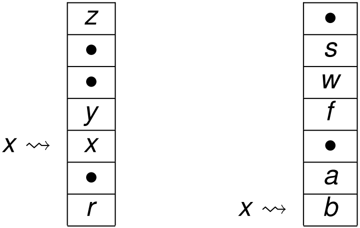
Similar story for linear probing.

Cuckoo hashing

Each key placed in one of two hash locations.



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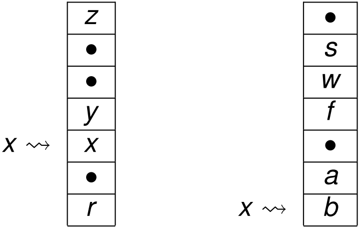

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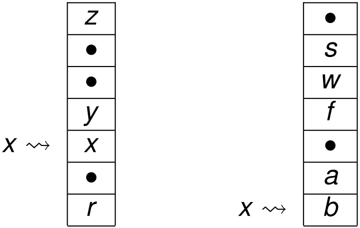

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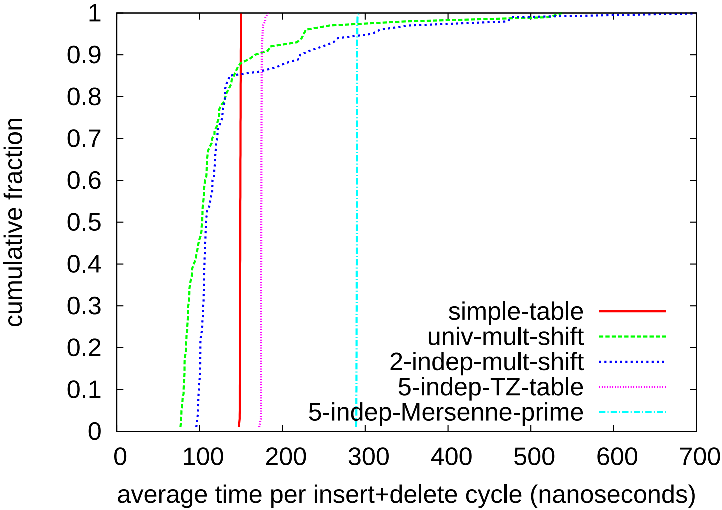
- Very delicate proof showing that obstruction can be used to code random tables R_i with few bits.

Speed

Hashing random keys		32-bit computer	64-bit computer
bits	hashing scheme	hashing time (ns)	
32	univ-mult-shift `(a*x)>>s`	1.87	2.33
32	2-indep-mult-shift	5.78	2.88
32	5-indep-Mersenne-prime	99.70	45.06
32	5-indep-TZ-table	10.12	12.66
32	simple-table	4.98	4.61
64	univ-mult-shift	7.05	3.14
64	2-indep-mult-shift	22.91	5.90
64	5-indep-Mersenne-prime	241.99	68.67
64	5-indep-TZ-table	75.81	59.84
64	simple-table	15.54	11.40

Experiments with help from Yin Zhang.

Robustness in linear probing for dense interval



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- Multiplicative hashing used in practice, but turns out to be very unreliable under typical denial-of-service (DoS) attacks based on consecutive IP addresses: systematic good performance 95% of the time, but systematic terrible performance 5% of the time [TZ'10].

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- Linear probing had gotten a reputation for being fastest in practice, but sometimes unreliable needing special protection against bad cases.
- Here we proved linear probing safe with good probabilistic performance for all input if we use simple tabulation.
- Simple tabulation also powerful for chaining, cuckoo hashing, and min-wise hashing:

  one simple and fast scheme for (almost) all your needs.

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- With chaining and linear probing, each operation takes expected constant time, but out of $\sqrt{n}$ operations, some are expected to take $\tilde{\Omega}(\log n)$ time.

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- With simple tabulation, additive term $(\max_i p_i)^{\gamma}$
  —in the hash tables we had $p \approx 1/n$.

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- Most people believe that deterministic constant time is not possible without randomization, but nobody can prove it.

Open problems

- Take any application using abstract truly random hash function, and prove that simple/twisted tabulation works.
- Could this be the first implementable hash function/RNG making classic quick sort work directly: using hash of i to generate index of ith pivot?
- Hash tables are used to look up keys in a dynamic set of stored keys. Can this be done without randomization?
- Can we both insert and look up keys in constant deterministic time? (not just with high probability)
- Currently, the best answer is that we can do both in $O(\sqrt{\log n / \log\log n})$ worst-case time [Andersson Thorup STOC'00] —tight for more general predecessor problem.
- Most people believe that deterministic constant time is not possible without randomization, but nobody can prove it.
- So far, no technique is known that can make any such separation between deterministic and randomized solutions for *any* data structure problem.