An improved 2-agent kidney exchange mechanism

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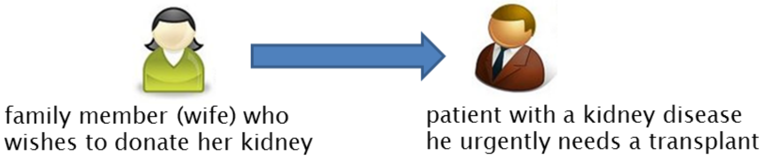
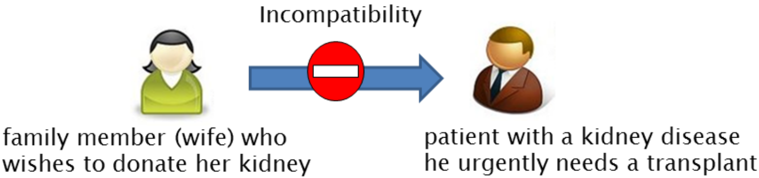
Pairwise kidney exchange

patient with a kidney disease
he urgently needs a transplant

Pairwise kidney exchange



Pairwise kidney exchange

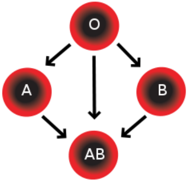


Compatibility

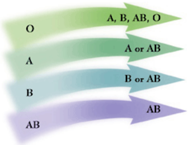
- Blood type compatibility
- Tissue type compatibility

Blood type compatibility

- Proteins A and B: Four different blood types.
- Compatibility: The donor does not introduce a new protein to the patient's blood.



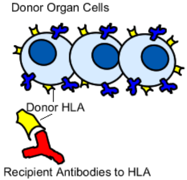
The 4 different blood types

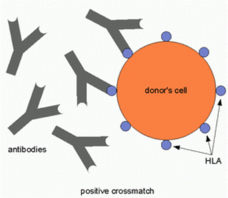


Blood type compatibilities

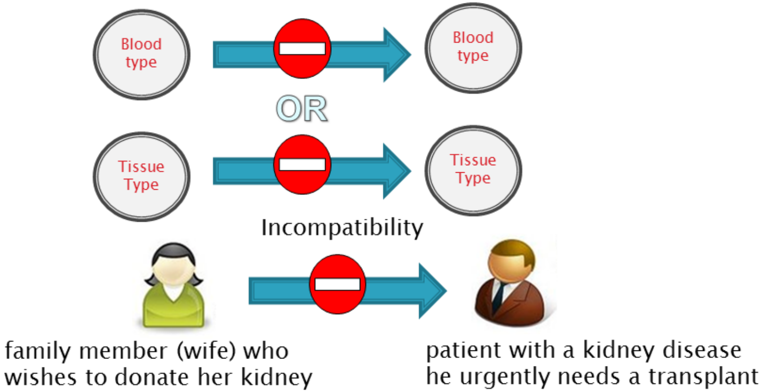
Tissue type compatibility

- Human Leukocyte Antigens (HLAs).
- Patient Antibodies.
- Positive Crossmatch.

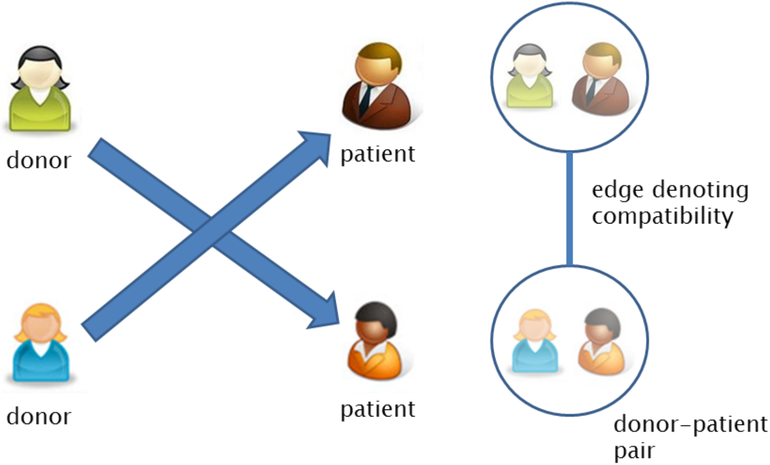




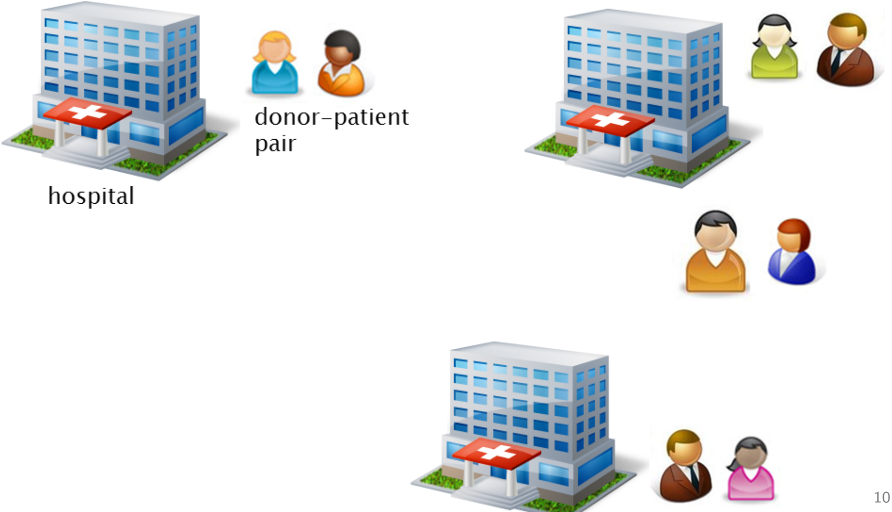
Pairwise kidney exchange



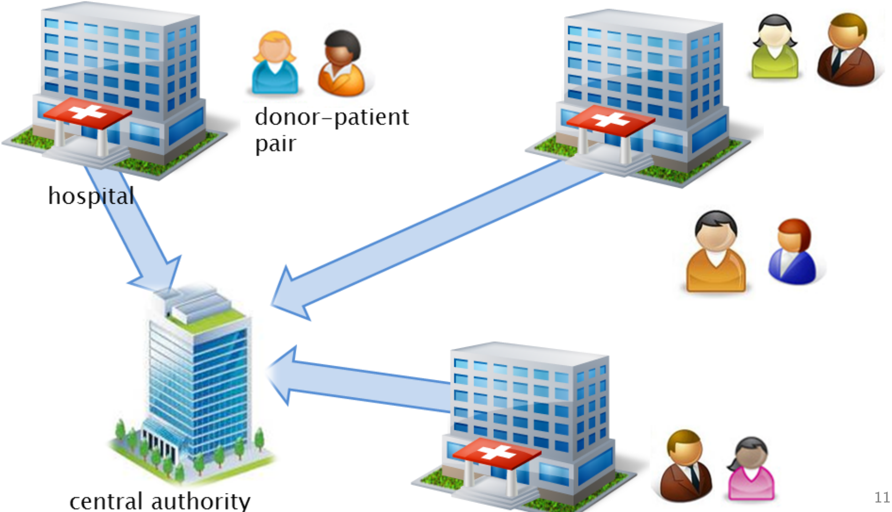
Pairwise kidney exchange



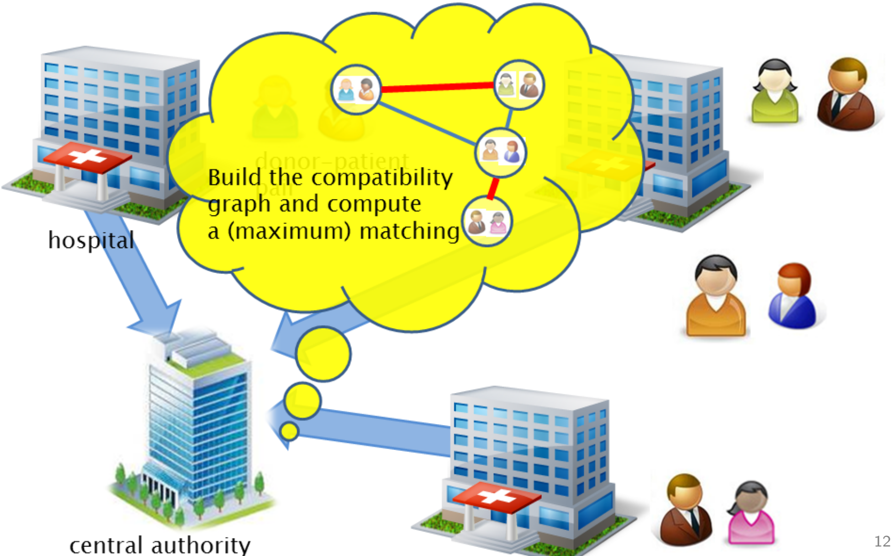
National kidney exchange programs



National kidney exchange programs



National kidney exchange programs



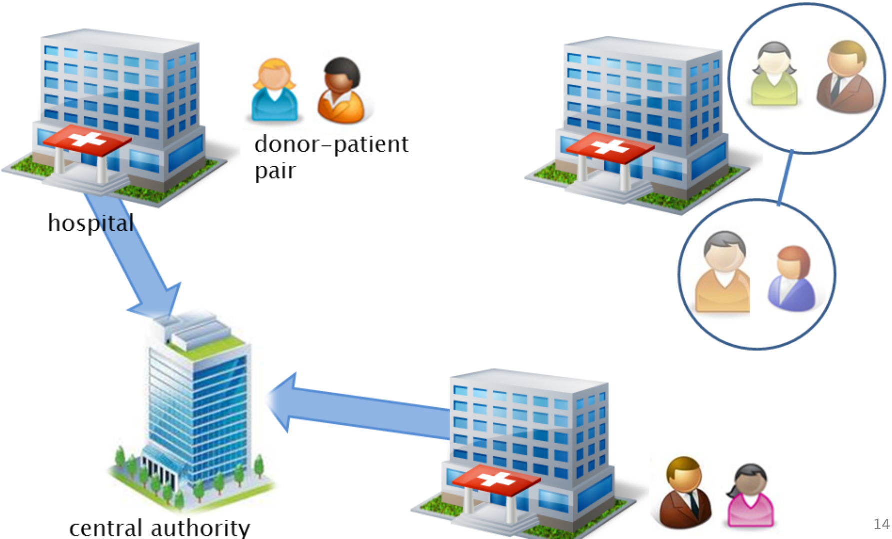
National kidney exchange programs

- Each patient-donor pair enrolls into a hospital.
- Each hospital collects the data and supplies them to the central authority.
- The central authority
  - Collects the data.
  - Constructs the compatibility graph.
  - Computes the maximum matching.

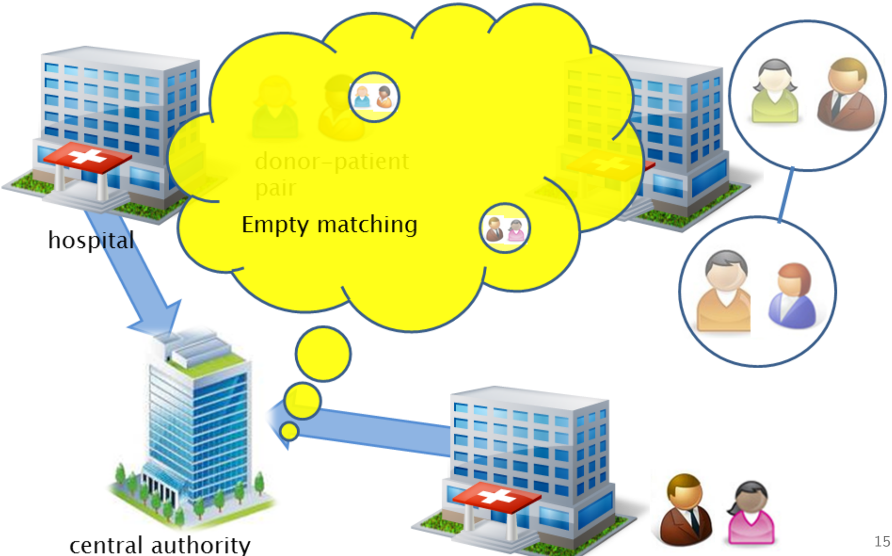
- So what is the problem?

- Incentives of hospitals
  - Each hospital aims to maximize the number of its donor-patient pairs that are matched.

National kidney exchange programs



National kidney exchange programs



Truthfulness: A requirement for kidney exchange mechanisms

- The input $x = (x_1, x_2, ...x_n)$ is provided by n self-interested agents(hospitals) and defines a compatibility graph.
- A mechanism A outputs a matching $A(x)$ on this graph.
- $\mathtt{gain}_i(A(x))$: The number of nodes of agent i that are incident to edges of $A(x)$.
- $\mathtt{gain}_i(A(t, x_{-i}))$: The total gain of agent i (including nodes matched internally) when reporting t as an input.
- A deterministic mechanism is truthful if $\mathtt{gain}_i(A(x)) \geqslant \mathtt{gain}_i(A(t, x_{-i}))$ for every agent i and every misreported input t.

Truthfulness: A requirement for kidney exchange mechanisms

- Randomized mechanisms
  - Probability distribution over deterministic mechanisms.
- A randomized mechanism A is
  - **Universally truthful**, when all deterministic mechanisms are truthful.
  - **Truthful in expectation** when

$$E[\mathtt{gain}_i(A(x))] \geqslant E[\mathtt{gain}_i(A(t, x_{-i}))].$$

Maximum matching is not truthful

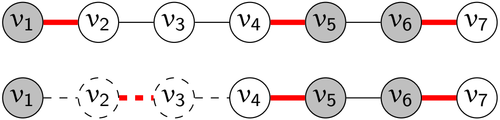


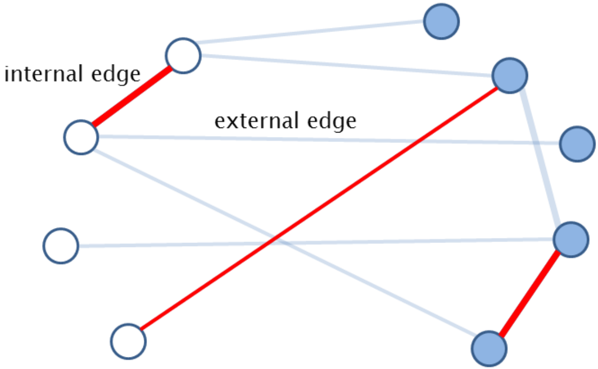
Figure : Agent 1 hides nodes v_2 and v_3 and matches them internally. The maximum matching on the new instance contains two white nodes of agent 1.

Efficiency

- A mechanism is ρ-efficient if the matching it outputs has (expected) cardinality at most ρ times smaller than the cardinality of the maximum matching.
- We aim at designing ρ-approximate mechanisms with ρ as small as possible.
- Approximate mechanism design without money.

MATCH: A 2-approximate kidney exchange mechanism for 2 agents

- Proposed by *Ashlagi, Fischer, Kash & Procaccia*.
- Among all matchings with maximum number of internal edges, select one with the maximum number of external edges.



MATCH: A 2-approximate kidney exchange mechanism for 2 agents

- Among all matchings with maximum number of internal edges, select one with the maximum number of external edges.
- Truthfulness: Intuitively, the mechanism simulates the incentives of agents.
- 2-approximate: The matching returned is inclusion-maximal.

Related work

- Ashlagi, Fischer, Kash & Procaccia.
  - MATCH
  - MATCH_{Π}: Deterministic truthful mechanism for any number of agents that does not have a finite approximation ratio.
  - MIX-AND-MATCH: 2-approximate, randomized, universally truthful mechanism for any number of agents.
  - FLIP-AND-MATCH: 4/3-approximate, randomized mechanism for two agents. Conjectured to be truthful in expectation.
  - Lower bounds of 2 and 8/7 for deterministic truthful mechanisms and randomized truthful mechanisms respectively.
- Ashlagi & Roth (EC11).
- Toulis & Parkes (EC11).
- Many recent papers in Economics literature.

Our results

- An observation: FLIP-AND-MATCH is not truthful.
- WEIGHT-AND-MATCH: A new truthful 3/2-approximate mechanism for two agents.
- Lower bounds for mechanisms that are:
  - Truthful in expectation or universally truthful.
  - Inclusion-maximal or not.

- Focus on two agents:
  - Necessary in order to understand the general problem.
  - Ad hoc kidney exchange programs between hospitals.

Flip-and-Match

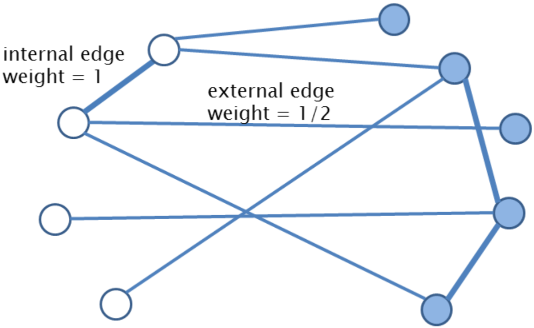
- Select equiprobably among
  - The matching returned my Match.
  - A maximum cardinality matching.
- The mechanism is 4/3-approximate.
- Is it truthful?
  - We prove that the mechanism is not truthful.
  - The problem seems to lie in the use of the maximum cardinality matching.

Our mechanism: WEIGHT-AND-MATCH

- Assign weights to the edges of the graph.
  - `weight` $= 1$ to the internal edges.
  - `weight` $= 1/2$ to the external edges.
- Compute two maximum weight matchings:
  - One with minimum cardinality (identical to MATCH).
  - One with maximum cardinality.
- Select equiprobably among the two matchings.

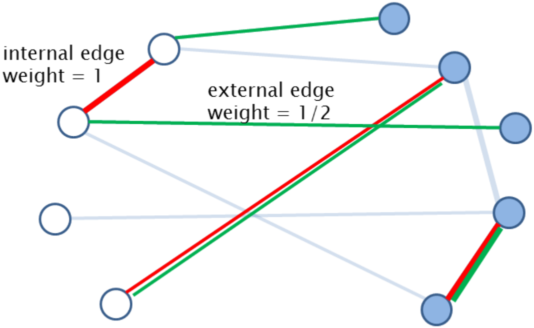
Our mechanism: WEIGHT-AND-MATCH

- Assign weights:



Our mechanism: WEIGHT-AND-MATCH

- Two matchings (red/green) with maximum weight 2.5:



Our mechanism: Weight-and-Match

Weight-and-Match

- Is 3/2-approximate.
- Can be executed in polynomial time.
- Is truthful in expectation.

Approximation Ratio

- WEIGHT-AND-MATCH is 3/2-approximate

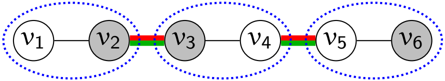


Figure : WEIGHT-AND-MATCH returns a matching of cardinality 2 while the maximum matching has cardinality 3.

Execution in polynomial time

- Slight modification of the edge weights.
- Maximum cardinality: We add a small quantity ϵ to the weight of each external edge.
- Minimum cardinality: We substract a small quantity ϵ from the weight of each external edge.
- We compute maximum weight matchings on the new instances.

Proving truthfulness

- Let M_1 and M_2 be the two matchings returned by the mechanism on input graph G.
- Assume that agent 1 hides some nodes and matches them internally.
- Let G' be the induced graph that does not contain the hidden nodes (and the edges incident to them).
- The mechanism returns two matchings on input graph G'.
- Let M_3 and M_4 be those two matchings augmented by the edges agent 1 uses to match the hidden nodes internally.

- Analysis: Compare M_1 to M_3 and M_2 to M_4.

Proving truthfulness

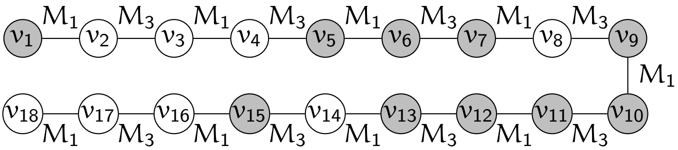
- Consider the symmetric differences of the compared matchings.
- Compare M_1 to M_3
  - $\texttt{gain}(M_3) = \texttt{gain}(M_1) - 2(\texttt{wgt}(M_1) - \texttt{wgt}(M_3))$
- Compare M_2 to M_4
  - $\texttt{gain}(M_4) \leqslant \texttt{gain}(M_2) + 2(\texttt{wgt}(M_2) - \texttt{wgt}(M_4))$
- Observe that $\texttt{wgt}(M_1) = \texttt{wgt}(M_2)$ and $\texttt{wgt}(M_3) = \texttt{wgt}(M_4)$.
- **Hence:** $\texttt{gain}(M_3) + \texttt{gain}(M_4) \leqslant \texttt{gain}(M_1) + \texttt{gain}(M_2)$.

$\mathtt{gain}(M_3) = \mathtt{gain}(M_1) - 2(\mathtt{wgt}(M_1) - \mathtt{wgt}(M_3))$

- Consider the quantities $n_{\mathtt{ww}}(M), n_{\mathtt{wg}}(M)$ and $n_{\mathtt{gg}}(M)$.
- Consider the symmetric difference $M_1 \Delta M_3 = (M_1 \backslash M_3) \cup (M_3 \backslash M_1)$.
- Connected components C, either cycles or paths.
- It suffices to prove $\mathtt{gain}(C_3) = \mathtt{gain}(C_1) - 2(\mathtt{wgt}(C_1) - \mathtt{wgt}(C_3))$ for each component.
- It holds that:
  - $\mathtt{gain}(M) = 2n_{\mathtt{ww}}(M) + n_{\mathtt{wg}}(M)$
  - $\mathtt{wgt}(M) = n_{\mathtt{ww}}(M) + \frac{1}{2}n_{\mathtt{wg}}(M) + n_{\mathtt{gg}}(M)$
- We prove that:
  - $n_{\mathtt{gg}}(M_1) = n_{\mathtt{gg}}(M_3)$

$n_{gg}(M_1) = n_{gg}(M_3)$

- C contains a block of t consecutive gray nodes $b_1, ..., b_t$.
- We argue that t can not be even.
- Assume that it is:
  - M_1 contains $\frac{t}{2} - 1$ edges $((b_2, b_3), (b_4, b_5), ..., (b_{t-2}, b_{t-1}))$ and M_3 contains $\frac{t}{2}$ edges $((b_1, b_2), (b_3, b_4), ..., (b_{t-1}, b_t))$.
  - By replacing the edges of M_1 with those of M_3 we aquire a matching with greater weight or equal weight and minimum cardinality.



- Summing over all components we prove $n_{gg}(M_1) = n_{gg}(M_3)$.

$\mathtt{gain}(M_4) \leqslant \mathtt{gain}(M_2) + 2(\mathtt{wgt}(M_2) - \mathtt{wgt}(M_4))$

- Consider the quantities $n_{\mathtt{ww}}(M), n_{\mathtt{wg}}(M)$ and $n_{\mathtt{gg}}(M)$.
- Consider the symmetric difference $M_2 \Delta M_4 = (M_2 \backslash M_4) \cup (M_4 \backslash M_2)$.
- Connected components C, either cycles or paths.

- We perform case analysis.

Lower bounds

- 5/4 for mechanisms that are truthful in expectation.
- 4/3 for mechanisms that are truthful in expectation and inclusion-maximal.
- 3/2 for mechanisms that are universally truthful.
- 2 for mechanisms that are universally truthful and inclusion-maximal.
  - Together with WEIGHT-AND-MATCH this indicates a separation between universal truthfulness and truthfulness in expectation.

A simulation

- Blood types are drawn from a national distribution.
- Tissue Type compatibility:
  - PRA: The probability that a patient will be tissue-type incompatible with a random donor.
  - We assume such incompatibilities occur independently and with equal probability.
- Create the random compatibility graph.
- Run WEIGHT-AND-MATCH (and MIX-AND-MATCH) on this graph.
- For all (but very small) input sizes, the mechanisms' approximation ratio is close to 1.

Open problems

- Close the gap between our upper and lower bounds.
  - Is there a 4/3-approximate inclusion-maximal truthful mechanism?
- Extend the upper bound for the case of many agents.
- Is there a deterministic truthful mechanism with finite approximation ratio?
- More general models
  - Cycles
  - Chains with altruists
- Dynamic models

Thank you very much