Single-Call Mechanisms

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joint work with Balasubramanian Sivan

Truthfulness:

A mechanism is *truthful* if a player's best strategy is to tell the truth.

Example:

A second-price auction is truthful because a player's best strategy is to bid the amount of money he is willing to pay.

Truthfulness requires information:

A bidder maximizes his utility, so truthful payments "balance" the bidder's utility across different outcomes.

...but how many outcomes are necessary to compute payments?

Single-call mechanisms are a powerful tool:

Given any procedure with truthful prices, *a single-call mechanism guarantees truthfulness using only the actual outcome!*

...with caveats.

Babaioff, Kleinberg, and Slivkins (2010):

Single-call mechanisms exist for monotone, single-parameter allocation procedures.

W and Sivan:

Single-call mechanisms exist using VCG prices.

What do single-call mechanisms look like?

What are the best single-call mechanisms?

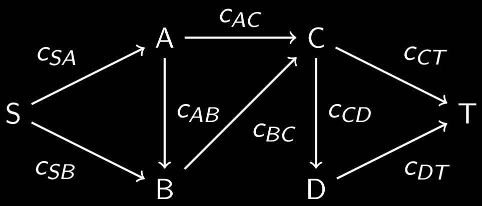
Nisan and Ronen (1998):

Naïvely computing VCG prices for a shortest-path auction requires the length of $|E| + 1$ different $S \to T$ paths.

VCG Shortest-Path Auctions

Standard model:

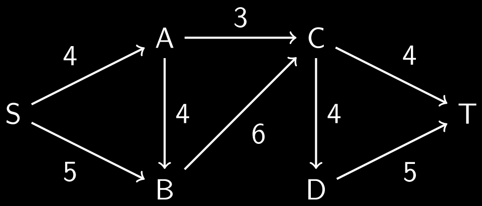
- Graph $G = (V, E)$.
- Edge $e \in E$ has cost c_e if used, 0 otherwise.
- c_e is private information.
- Auctioneer pays $-P_e$ to learn c_e.



VCG Shortest-Path Auctions

How much should we pay edge CT to learn c_{CT}?

$-P^{VCG}_{CT}$ = [Cost to others without CT] − [Cost to others with CT]



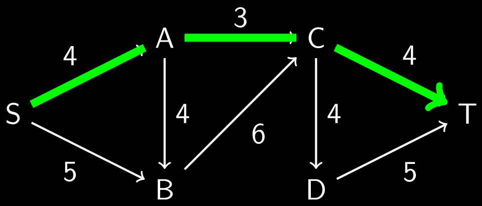
$$-P^{VCG}_{CT} =$$

VCG Shortest-Path Auctions

How much should we pay edge CT to learn c_{CT}?

$$-P_{CT}^{VCG} = [\text{Cost to others without CT}] - [\text{Cost to others with CT}]$$

Shortest $S \to T$ path



$-P_{CT}^{VCG} = \quad - cost(SAC) = \quad -7$

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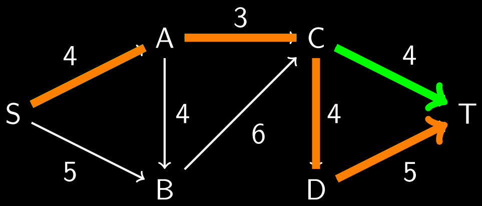
Shortest $S \to T$ path

Shortest $S \to T$ path without edge CT



$$-P_{CT}^{VCG} = cost(SACDT) - cost(SAC) = 16 - 7 = 9$$

VCG Shortest-Path Auctions

Algorithm 1: Naïve auction for $S \to T$ paths in $G = (V, E)$

input : Bids $b_e = c_e$.

output : $S \to T$ path and payments $\{P_e\}$.

for $e \in E$ **do**

  $\Gamma'_e \leftarrow$ shortest path in $G' = (V, E \setminus \{e\}, c_e)$;

$\Gamma \leftarrow$ shortest path in $G = (V, E, c_e)$;

outcome : Γ

payments: $-P_e = \sum_{x \in \Gamma'_e} c_x - \sum_{x \in \Gamma \setminus \{e\}} c_x$

What if we can only compute the length one path?

Babaioff, Kleinberg, and Slivkins (2010):

Truthfulness in expectation only requires measurements along the path you actually use!

...if you occasionally pick the wrong one...

...and you occasionally get large rebates.

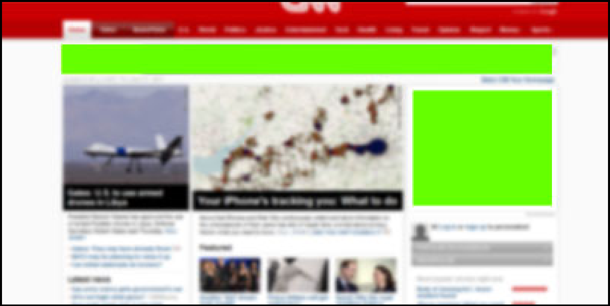
Why do we care?

Application:

PPC Advertising Auctions

PPC Ad Auctions

I have two ad slots to sell to two bidders:



Pay-Per-Click (PPC) — bidders only pay when ad is clicked.

PPC Ad Auctions

Standard model:

- Ad slots $j \in \{1, 2\}$, probability user clicks (CTR) is $c_1 > c_2$
- Bidders $i \in \{1, 2\}$, value-per-click v_i where $v_1 > v_2$
  - Utility $\mathbf{E}[u_i] = c_{j(i)} v_i - P_i$
- *Auctioneer estimates* of CTRs $\hat{c}_j$

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True VCG price:

$$\mathbf{E}[P_1^{VCG}] = (c_1 - c_2) v_2$$

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Pay-per-click (PPC) price using estimates:

$$PPC_1 = \frac{1}{\hat{c}_1} (\hat{c}_1 - \hat{c}_2) v_2$$

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Expected PPC price:

$$\mathbf{E}[P_1] = c_1 PPC_1 = \frac{c_1}{\hat{c}_1}(\hat{c}_1 - \hat{c}_2) v_2 \left(\neq P_1^{VCG}\right)$$

PPC Ad Auctions

Nontruthful Example:

Auction parameters:

$$v_1 = \$1.10 \qquad c_1 = 0.1$$
$$v_2 = b_2 = \$1 \qquad \hat{c}_1 = 0.11$$
$$c_2 = \hat{c}_2 = 0.09$$

Advertiser 1 prefers to lie:

Bid:	$b_1 = v_1 = \$1.10$		$b_1 = \$0$
Expected Utility:	0.092	$<$	0.099

PPC Ad Auctions

Single-Call Solution:

We can use the BKS single-call construction:

- Even bad estimates $\hat{c}_j$ give a monotone, single-parameter allocation procedure.
- Observing clicks gives an unbiased estimate of a bidder's true expected value $c_{j(i)} v_i$.

Informational limitations are general...

PPC advertising auctions with estimated CTRs:

$$[value] = [bid] \times [CTR]$$

Machine scheduling with per-unit-time bids:

$$[cost] = [bid] \times [runtime]$$

$$\vdots$$

Any allocation procedure where bids are “incomplete” could be vulnerable!

Single-call mechanisms offer robust truthfulness guarantees.

Single-Call Mechanisms with VCG Prices:
VCG Shortest-Path Example

VCG Shortest-Path Auctions

How much should we pay edge CT to learn c_{CT}?

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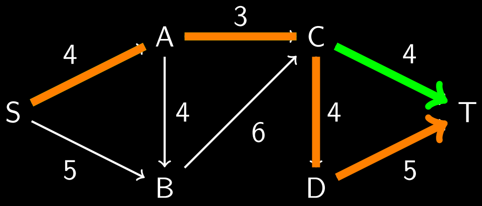


$$-P^{VCG}_{CT} = cost(SACDT) - cost(SAC) = 16 - 7 = 9$$

VCG Shortest-Path Auctions

Problem:

We observe $cost(SAC)$, but $cost(SACDT)$ could be wrong!

VCG Shortest-Path Auctions

Idea:

Most of the time...

...select $SACT$ and pay $-P_{CT} \approx -cost(SAC)$.

With small probability γ...

...select $SACDT$ and pay rebate $-P_{CT} \approx \frac{1}{\gamma}cost(SACDT)$.

Result:

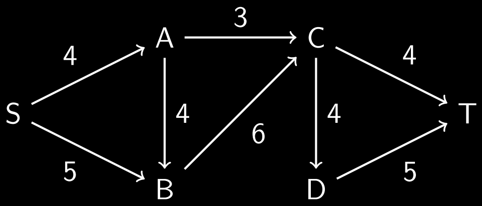
$$-\mathbf{E}[P_{CT}] \approx cost(SACDT) - cost(SAC)$$

and we only needed observed costs!

VCG Shortest-Path Auctions

A full single-call implementation:

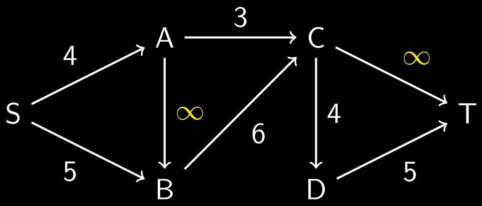
1. Start with given costs.



VCG Shortest-Path Auctions

A full single-call implementation:

1. Start with given costs.
2. Randomly change some costs to ∞.



VCG Shortest-Path Auctions

A full single-call implementation:

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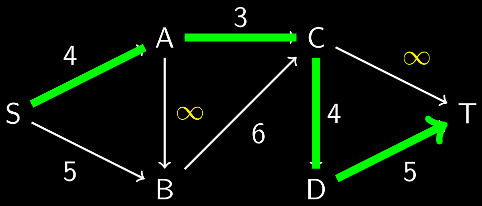
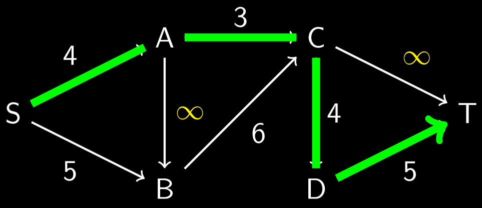


VCG Shortest-Path Auctions

A full single-call implementation:

1. Start with given costs.
2. Randomly change some costs to ∞.
3. Compute shortest path.
4. Pay edge e

$$-\mathcal{P}_e = \left(\sum_{x\in\Gamma\setminus\{e\}} c_x\right) \times \begin{cases} -1, & \hat{c}_e = c_e \\ \frac{1-\gamma}{\gamma}, & \hat{c}_e = \infty \end{cases}$$



VCG Shortest-Path Auctions

Algorithm 2: Single-call auction for $S \to T$ paths in $G = (V, E)$

input : Bids $b_e = c_e$.

output : $S \to T$ path and payments $\{P_e\}$.

for $e \in E$ **do**

  with probability $1 - \gamma$

    $\hat{c}_e \leftarrow c_e$;

  otherwise

    $\hat{c}_e \leftarrow \infty$;

$\Gamma \leftarrow$ shortest path in $G = (V, E, \hat{c}_e)$;

outcome : Γ

payments: $-\mathcal{P}_e = \left(\sum_{x \in \Gamma \setminus \{e\}} c_x\right) \times \begin{cases} -1, & \hat{c}_e = c_e \\ \frac{1-\gamma}{\gamma}, & \hat{c}_e = \infty \end{cases}$

Single-Call Theory

What is a Mechanism?

Players bid:

For each possible outcome o, a player claims to have value $b_i(o)$.

(his actual value is $v_i(o)$)

The "auctioneer" picks an outcome:

Model as an *allocation function* A mapping bids b to outcomes o.

(e.g. $A(b) =$ "give the item slot to the highest bidder.")

Money exchanged:

Bidders pay P_i to auctioneer.

Single-Call Mechanisms

Mechanism $\mathcal{M} = (A, \{\mathcal{P}_i\})$ that is truthful in expectation and calls A only once.

Single-Call ~~Mechanisms~~ Reductions

Input:

Black box allocation rule A that maximizes welfare.

Output:

Mechanism $\mathcal{M} = (A, \{\mathcal{P}_i\})$ that is truthful in expectation and calls A only once.

Single-Call ~~Mechanisms~~ Reductions

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Allocation rule $\mathcal{A} \approx A$ *and*
mechanism $\mathcal{M} = (\mathcal{A}, \mathcal{P}_i)$ that is truthful in expectation.

Single-call reductions are the tool for creating single-call mechanisms.

Prior Work

Theorem (Babaioff, Kleinberg, and Slivkins 2010)

There is a single-call reduction that takes as input any monotone single-parameter allocation rule $A(b)$ and returns a truthful-in-expectation mechanism $\mathcal{M} = (\mathcal{A}, \{\mathcal{P}_i\})$ with $\mathcal{A} \approx A$ that only calls A once.

Our Work

W and Sivan:

BKS works for single-parameter domains...does a similar approach work using VCG prices?

There is a single-call reduction for MIDR allocation rules that uses VCG prices.

What do single-call mechanisms look like?

We characterize "all single-call reductions" for single-parameter domains and VCG prices.

Large rebates and "wrong" choices are bad...

BKS and WS are optimal for their domains.

Existence:

"VCG" Single-Call Reductions

The $\texttt{MIDRtoMech}(A, \gamma)$ Reduction

Algorithm 3: $\texttt{MIDRtoMech}(A, \gamma)$

input : MIDR allocation function A.

output: Truthful-in-expectation mechanism $\mathcal{M} = (\mathcal{A}, \{\mathcal{P}_i\})$.

Solicit bids b from agents;

for $i \in [n]$ **do**

  with probability $1 - \gamma$

    Set $\hat{b}_i = b_i$;

  otherwise

    Set $\hat{b}_i =$ "*zero*";

Outcome: $\mathcal{A}(b) = A(\hat{b})$;

Payments:

$$\mathcal{P}_i(b) = \left(\sum_{j \neq i} b_j(A(\hat{b}))\right) \times \begin{cases} -1, & \hat{b}_i = b_i \\ \frac{1-\gamma}{\gamma}, & \hat{b}_i = \text{“}zero\text{”} \end{cases};$$

The $\texttt{MIDRtoMech}(A, \gamma)$ Reduction

Theorem

For all maximal-in-distributional-range (MIDR) allocation rules A, the single-call reduction $\texttt{MIDRtoMech}(A, \gamma)$ produces a mechanism $\mathcal{M} = (\mathcal{A}, \{\mathcal{P}_i\})$ that is truthful in expectation.

Characterization:

What Single-Call Reductions Exist?

Characterizing Reductions

A single-call reduction/mechanism must...

- Take A as a black box.
- Request b from bidders.
- Evaluate A on at most one bid vector $\hat{b}$, causing the outcome $A(\hat{b})$ to be realized.
- Charge payments $\mathcal{P}_i$ that are a function of things it knows.

Characterizing Reductions

Algorithm 4: Generic Single-Call Reduction $(\mu, \{\mathcal{P}_i\})$

input : Black box access to allocation function A.
output : Truthful-in-expectation mechanism $\mathcal{M} = (\mathcal{A}, \{\mathcal{P}_i\})$.

Solicit bid vector b from agents;

Sample $\hat{b} \sim \mu_b$;

outcome : $\mathcal{A}(b) = A(\hat{b})$
payments: $\mathcal{P}_i(A(\hat{b}), \hat{b}, b)$

Characterizing VCG-Based Reductions

Theorem

A single-call reduction $(\mu, \{\mathcal{P}_i\})$ using VCG prices is truthful for all MIDR allocation rules A if and only if:

- μ *corresponds to the following idea:* randomly pick a set of bidders $M \subseteq [n]$ and "ignore" bidders not in M. *The distribution* $\Pr(M)$ *should be independent of b*
- $\mathcal{P}_i = \left(\sum_{j \neq i} b_j(A(\hat{b}^M))\right) \times \begin{cases} -1, & i \in M \\ \frac{\Pr(M \cup \{i\})}{\Pr(M)}, & i \notin M \end{cases}$

Proof idea:

- μ must be such that $\mathcal{A}$ is MIDR.
- Write VCG payments for general single-call reduction and equate to $\mathcal{P}$.

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Optimality:

What are the Best Single-Call Reductions?

Single-call reductions trade *expectation* for *risk*.

Optimal Reductions: Expectation

Single-call mechanisms sacrifice expected behavior:

- Expected welfare may be worse than A
- Expected revenue may be less than A.
- Nontrivial likelihood of choosing "wrong" outcome. (Precision)

Optimal Reductions: Risk

Single-call payments may have a high variance:

- Large payment variance $\max \mathrm{Var}(\mathcal{P}_i)$
- Large worst-case payment $\max |\mathcal{P}_i|$

Optimal Reductions

Lemma

The reduction $\texttt{MIDRtoMech}(A, \gamma)$...

- ...*has (normalized) payment variance* $\frac{1-\gamma}{\gamma}$.
- ...*has (normalized) worst-case payment* $\frac{1-\gamma}{\gamma}$.
- ...*guarantees a welfare approximation of* $1-\gamma$.
- ...*guarantees a revenue approximation of* $(1-\gamma)^n$.
- ...*has precision* $(1-\gamma)^n$.

Optimal Reductions

Theorem (Optimality of $\texttt{MIDRtoMech}(A, \gamma)$)

Among single-call reductions that yield a truthful mechanism for all MIDR A, $\texttt{MIDRtoMech}(A, \gamma)$ *simultaneously optimizes* $\max \mathrm{Var}(\mathcal{P}_i)$ *and* $\max |\mathcal{P}_i|$ *with respect to a bound on welfare, revenue, or precision.*

Single-Parameter Domains

Single-Parameter Domains

Characterization:

Analogous characterization holds for single-parameter domains, but a general proof requires measure theory.

Proof uses payment characterization from Archer and Tardos (2001).

Optimality:

BKS optimizes metrics simultaneously for bidders with positive types.

Conclusion and Open Questions

Informational restrictions are natural and common:

Problems like PPC ad auctions have limited information that breaks truthfulness for standard truthful mechanisms.

Single-Call Mechanisms are powerful and general:

Single-call mechanisms can be constructed in many ways for two of the largest classes of allocations that admit truthful prices...

...but they have substantial tradeoffs.

Main open question:

What domains admit practical single-call mechanisms?

Thank you.