Quantum Algorithms for Quantum Field Theories

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Joint work with
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The full description of quantum mechanics for a large system with $R$ particles has too many variables. It cannot be simulated with a normal computer with a number of elements proportional to $R$.

-Richard Feynman, 1982

An $n$-bit integer can be factored on a quantum computer in $O(n^2)$ time.

-Peter Shor, 1994
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Are there any systems that remain hard to simulate even with quantum computers?
Quantum Simulation

Condensed-matter lattice models:

[Lloyd, 1996]
[Abrams, Lloyd, 1997]
[Berry, Childs, 2012]

Many-particle Schrödinger and Dirac Equations:

[Meyer, 1996]
[Zalka, 1998]
[Taylor, Boghosian, 1998]
Quantum Field Theory

- Much is known about using quantum computers to simulate quantum systems.
- Why might QFT be different?
  - Field has infinitely many degrees of freedom
  - Relativistic
  - Particle number not conserved
  - Formalism looks different
What is the computational power of our universe?
When do we need QFT?

Nuclear Physics

Accelerator Experiments

Cosmic Rays

> Whenever quantum mechanical and relativistic effects are both significant.
Classical Algorithms

Feynman diagrams

- Break down at strong coupling or high precision

Lattice methods

- Cannot calculate scattering amplitudes
A QFT Computational Problem

**Input:** a list of momenta of incoming particles

**Output:** a list of momenta of outgoing particles
I will present a polynomial-time quantum algorithms to compute scattering probabilities in the $\phi^4$ and Gross-Neveu models with nonzero mass.

These are simple models that illustrate some of the main difficulties in simulating a QFT:

- Discretizing spacetime
- Preparing initial states
\( \phi^4 \)-theory

Lagrangian density

\[
\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - m^2 \phi^2 - \frac{\lambda}{4!} \phi^4
\]

For quantum simulation we prefer Hamiltonian formulation (equivalent)

\[
H = \int d^d x \left[ \pi^2 + (\nabla \phi)^2 + m^2 \phi^2 + \lambda \phi^4 \right]
\]

\[
[\phi(x), \pi(y)] = i \delta^{(d)}(x - y)
\]
Quantum Fields

A classical field is described by its value at every point in space.

\[ E(r) = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \]

A quantum field is a superposition of classical field configurations.

\[ |\Psi\rangle = \int \mathcal{D}[E] \Psi[E] \ |E\rangle \]
A configuration of the field is a list of field values, one for each lattice site.

A quantum field can be in a superposition of different field configurations.
Particles Emerge from Fields

Particles of different energy are different resonant excitations of the field.
Lattice cutoff

Continuum QFT = limit of a sequence of theories on successively finer lattices
Coarse grain

Mass: \( m \)

Interaction strength: \( \lambda \)

Mass: \( m' \)

Interaction strength: \( \lambda' \)
Lattice cutoff

Continuum QFT = limit of a sequence of theories on successively finer lattices

$m$ and $\lambda$ are functions of lattice spacing!
Discretization Errors

- Renormalization of $m$ and $\lambda$ make discretization tricky to analyze
- In $\phi^4$-theory, in $d=1,2,3$, discretization errors scale as $\alpha^2$

\[ \begin{align*}
\int_0^\infty &\int_0^\infty \angle &\frac{(-i\lambda_0)^2}{6} \int \frac{d^D k}{(2\pi)^D} \int \frac{d^D q}{(2\pi)^D} \sum_i \frac{4}{a^2} \sin^2 \left( \frac{ak^i}{2} \right) - m^2 \frac{i}{(q^0)^2 - \sum_i \frac{4}{a^2} \sin^2 \left( \frac{aq^i}{2} \right) - m^2} \\
&\times (p^0 + k^0 + q^0)^2 - \sum_i \frac{4}{a^2} \sin^2 \left( \frac{a(p^i + k^i + q^i)}{2} \right) - m^2 \\
&= \frac{i\lambda_0^2}{3} \int_0^1 \int_0^1 \int_0^1 dx \ dy \ dz \ \delta(x + y + z - 1) \int \frac{d^D k}{(2\pi)^D} \int \frac{d^D q}{(2\pi)^D} \frac{1}{D^3} 
\end{align*} \]
Simulation converges as $a^2$
Condensed Matter

There is a fundamental lattice spacing.

But:

We may save qubits by simulating a coarse-grained theory.
After imposing a spatial lattice we have a many-body quantum system with a local Hamiltonian

Simulating the time evolution in polynomial time is a solved problem

Standard methods scale as $N^2$. We can do $N$. 

- Discretizing spacetime
- Preparing initial states
With particle interactions turned off, the model is exactly solvable.

1. Prepare non-interacting vacuum (Gaussian)
2. Prepare wavepackets of the non-interacting theory
3. Adiabatically turn on interactions
4. Scatter
5. Adiabatically return to non-interacting theory to make measurements
Adiabatically Turn on Interaction

\[ H(s) = \sum_{x \in \Omega} \left[ \pi^2 + (\nabla \phi)^2 + m^2 \phi^2 + s \lambda \phi^4 \right] \]

Use standard techniques to simulate \( H(s) \) with \( s \) slowly varying from 0 to 1

This *almost* works....
During the slow time evolution

The wavepackets propagate and broaden.
**Solution:** intersperse backward time evolutions with time-independent Hamiltonians

This winds back the dynamical phase on each eigenstate, without undoing the adiabatic change of eigenbasis.
**Strong Coupling**

$\phi^4$-theory in 1+1 and 2+1 dimensions has a quantum phase transition in which the $\phi \rightarrow -\phi$ symmetry is spontaneously broken.

Near the phase transition perturbation theory fails and the gap vanishes.

$$m_{\text{phys}} \sim (\lambda_c - \lambda_0)^\nu$$

$$\nu = \begin{cases} 
1 & d = 1 \\
0.63 \ldots & d = 2 
\end{cases}$$
Complexity

Weak Coupling:

\[
\begin{array}{|c|c|}
\hline
 d = 1 & \left(\frac{1}{\epsilon}\right)^{1.5} \\
\hline
 d = 2 & \left(\frac{1}{\epsilon}\right)^{2.376} \\
\hline
 d = 3 & \left(\frac{1}{\epsilon}\right)^{5.5} \\
\hline
\end{array}
\]

Strong Coupling:

\[
\begin{array}{|c|c|c|}
\hline
 & \lambda_c - \lambda_0 & p & n_{out} \\
\hline
 d = 1 & \left(\frac{1}{\lambda_c - \lambda_0}\right)^9 & p^4 & n_{out}^5 \\
\hline
 d = 2 & \left(\frac{1}{\lambda_c - \lambda_0}\right)^{6.3} & p^6 & n_{out}^{7.128} \\
\hline
\end{array}
\]
Fermions:

- Fermion doubling problem
- Free vacuum different from Bosonic case

Gross-Neveu:

\[ H = \int dx \left[ \sum_{j=1}^{N} \bar{\psi}_j \left( m_0 - i \gamma^1 \frac{d}{dx} \right) \psi_j + \frac{g^2}{2} \left( \sum_{j=1}^{N} \bar{\psi}_j \psi_j \right)^2 \right] \]
Fermion Doubling Problem

\[ \frac{d\psi}{dx} \rightarrow \frac{\psi(x + a) - \psi(x - a)}{2a} \]

\[ \sqrt{p^2 + m^2} \rightarrow \sqrt{\sin^2 p + m^2} \]
Wilson Term

\[ H \rightarrow H - \frac{r}{2a} \sum_x \bar{\psi} (\psi(x + a) - 2\psi(x) + \psi(x - a)) \]
Preparing Fermionic Vacuum

\[ H = \sum_x \bar{\psi}(x) m \psi(x) \]

\[ H = \sum_x \bar{\psi}(x) \left( m + \frac{d}{dx} \right) \psi(x) \]

\[ H = \sum_x \bar{\psi}(x) \left( m + \frac{d}{dx} \right) \psi(x) + \frac{g^2}{2} (\bar{\psi}(x)\psi(x))^2 \]
Eventual goal:
Simulate the standard model in polynomial time with quantum circuits.

Solved problems:
$\phi^4$-theory [Science, 336:1130 (2012)]
Gross-Neveu [S.J., Lee, Preskill, in preparation]

Open problems:
gauge symmetries, massless particles, spontaneous symmetry breaking, bound states, confinement, chiral fermions
Analog Simulation

- No gates: just implement a Hamiltonian and let it time-evolve

- Current experiments do this!
Broader Context

- Quantum field theories
- Topological quantum field theories
  - Turaev-Viro
  - HOMFLY
  - Ponzano-Regge
  - Jones
  - Tutte
- Quantum circuits
- Game trees
- Formula evaluation
- Scattering ("quantum walks")

jeudi 24 janvier 13
What I’m trying to do is get you people who think about computer simulation to see if you can’t invent a different point of view than the physicists have.

-Richard Feynman, 1981

In thinking and trying out ideas about “what is a field theory” I found it very helpful to demand that a correctly formulated field theory should be soluble by computer... It was clear, in the ‘60s, that no such computing power was available in practice.

-Kenneth Wilson, 1982
Conclusion

Quantum computers can simulate scattering in $\phi^4$-theory and the Gross-Neveu model.

There are many exciting prospects for quantum computation and quantum field theory to contribute to each other’s progress.

I thank my collaborators:

Thank you for your attention.