Computational Intractability and Asymmetric Information in Financial Derivatives

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Crash of ‘08 (in 1 slide)

Relative Market Sizes

Lesson Learnt

Small Error in Derivatives ➔ Huge Effects in Economy

Example of derivative

"Fair price" = $1M X Pr[DOW >11,000]

Derivative pricing can be hard

Contract

Seller to Pay Buyer $1M if DOW a year from today is FIRST FIVE digits of a factor of

2138746322342…(10000 digits)

"Fair price" =??

Computation requires factoring integers!

This talk: Similar intractability can arise in case of more common, less exotic derivatives

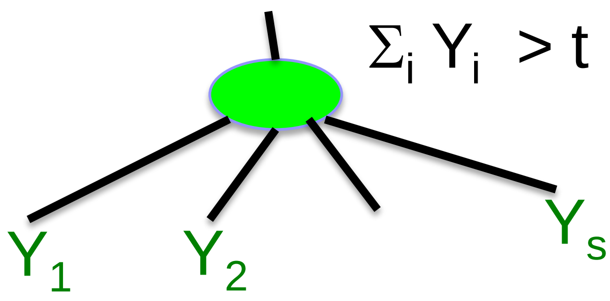
What is a financial derivative?

Some stochastic economic variables Y_1, Y_2,…, Y_s (stock price, DOW, prime rate, etc.;)

Payoff function $f(Y_1, Y_2,\ldots,Y_s)$.

"Fair Price" = $E[f(Y_1, Y_2,\ldots,Y_s)]$ (risk-neutral buyers)

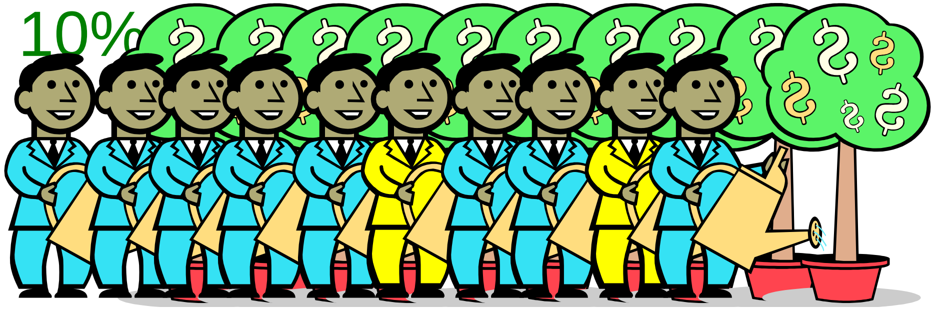
CDO: Y_1, Y_2,…,Y_s are payoffs of mortgages or another debt. Payoff iff sum of Y_i's exceeds some threshold.



CDO^2: CDO in which Y_1, Y_2,…,Y_s are themselves CDO payoffs.

CDOs:Simplistic explanation

Y_1, Y_2,…, Y_{100} : Mortgages of face value $1M;
default probability 10%



Expected total yield: $90M

Create two tranches: senior and junior.
Senior gets first $70M of yield; junior gets rest

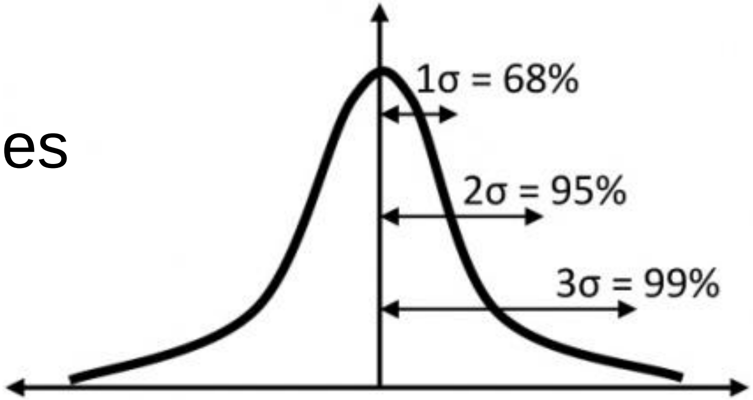
Important: Senior tranche attractive even if buyer believes 10 mortgages are "Lemons" ("asymmetric info")

Senior tranche less risky, attractive to pension funds etc.
Junior tranche more risky, attractive to hedge funds

Economists' belief: Derivatives "solve" the problem of asymmetric info (aka lemon problem) [DeMarzo-Duffie'99],[DeMarzo'05]

Law of large #s: pool yields are gaussian

Sum of D uniform iid 0/1 variables
= Gaussian with mean D/2 and
$\sigma = \frac{\sqrt{D}}{2}$



simplified "binary" version of tranching:
yield > threshold: senior tranche gets everything;
yield < threshold: senior tranche gets nothing.

threshold = D/2 - 3σ ➔ 1% default probability for senior tranche

(call this "3σ binary CDO", models credit downgrade risk)

Our res

As Hard as DensestSubgraph Problem

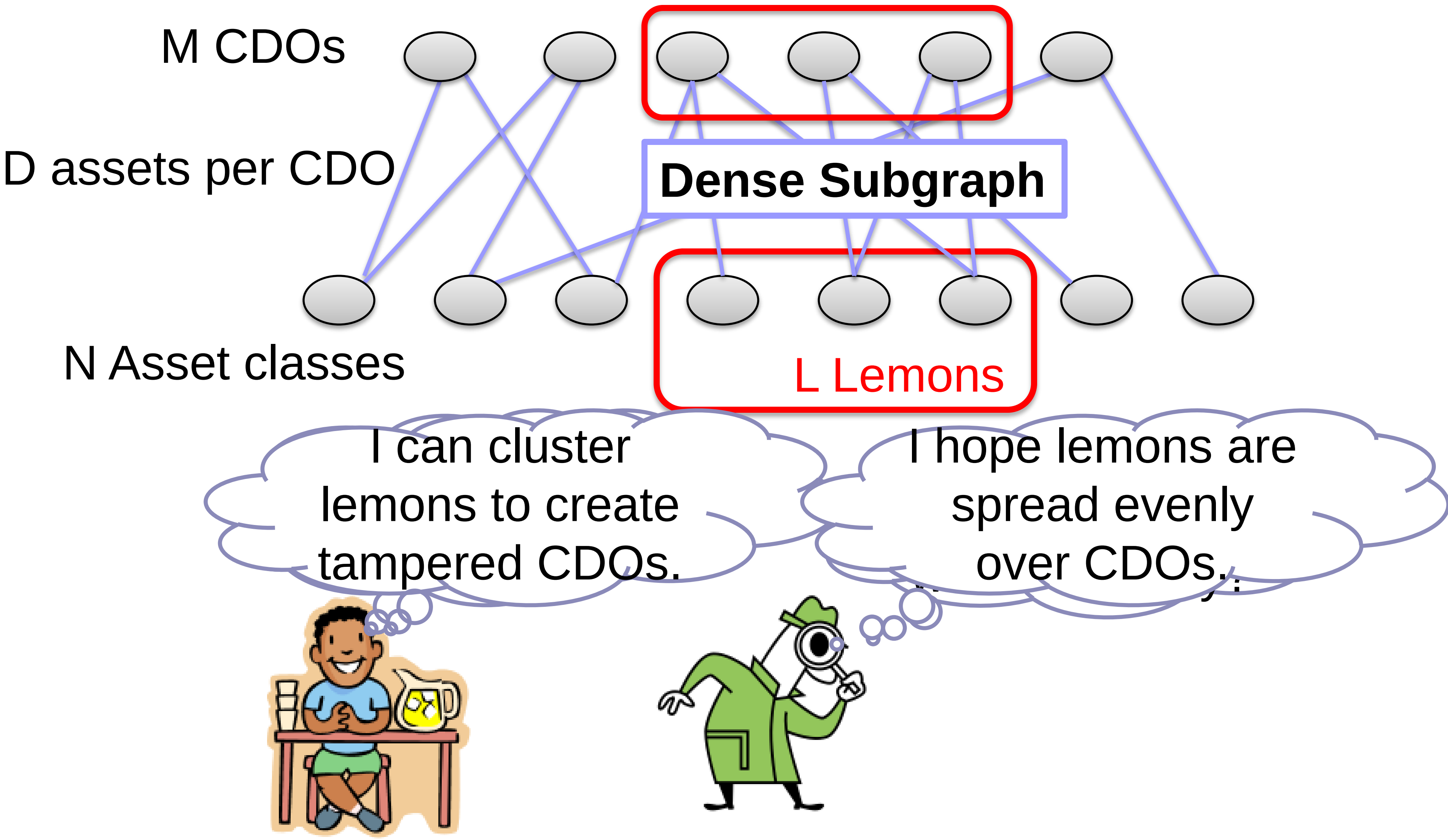
- Pricing can be computationally intractable for popular derivatives like CDOs.
  - Average Case Complexity

- Effects of asymmetric info (“lemon costs”) can persist or even amplify when buyers are computationally limited, whereas they → 0 for computationally unbounded buyers.

- Notion of “complexity lemon cost” can help distinguish different derivatives (eg CDO vs. CDO^2)
  - …. “Complexity ranking” (though incomplete)
  (Open problem in Brunnermeier-Oehmke 2009)

Simplest Model

6σ lemons, default w.p. ½



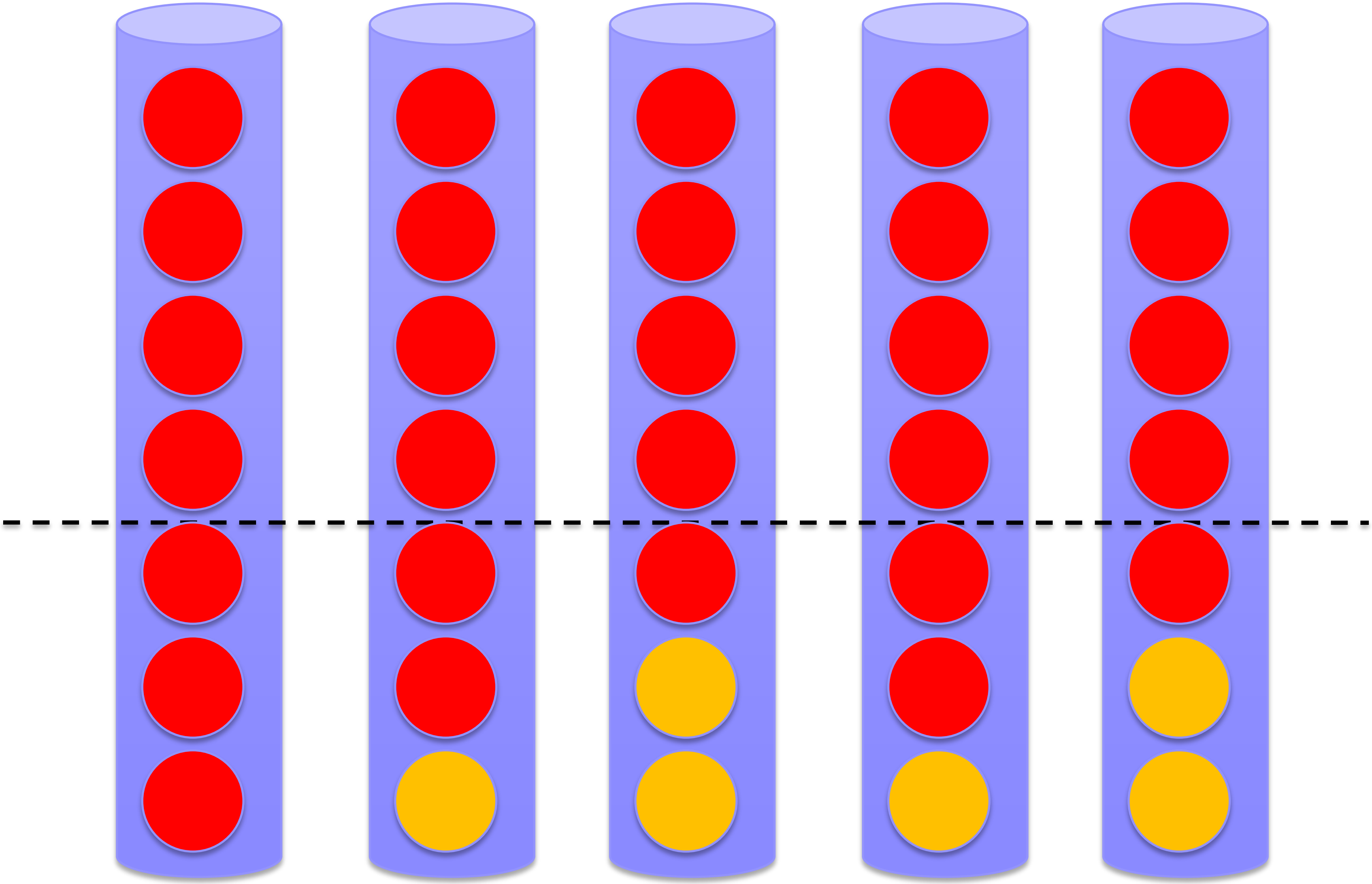
Thm 1: Seller can easily generate two distinct distributions D_1, D_2 on bundles of M 3σ–binary CDOs such that:

- D_1 = totally random bundle
- D_2 = Tampered bundle, each tampered CDO has > 6σ lemons.
- Polynomial time buyer cannot distinguish
D_2 with any reasonable chance(** reminiscent

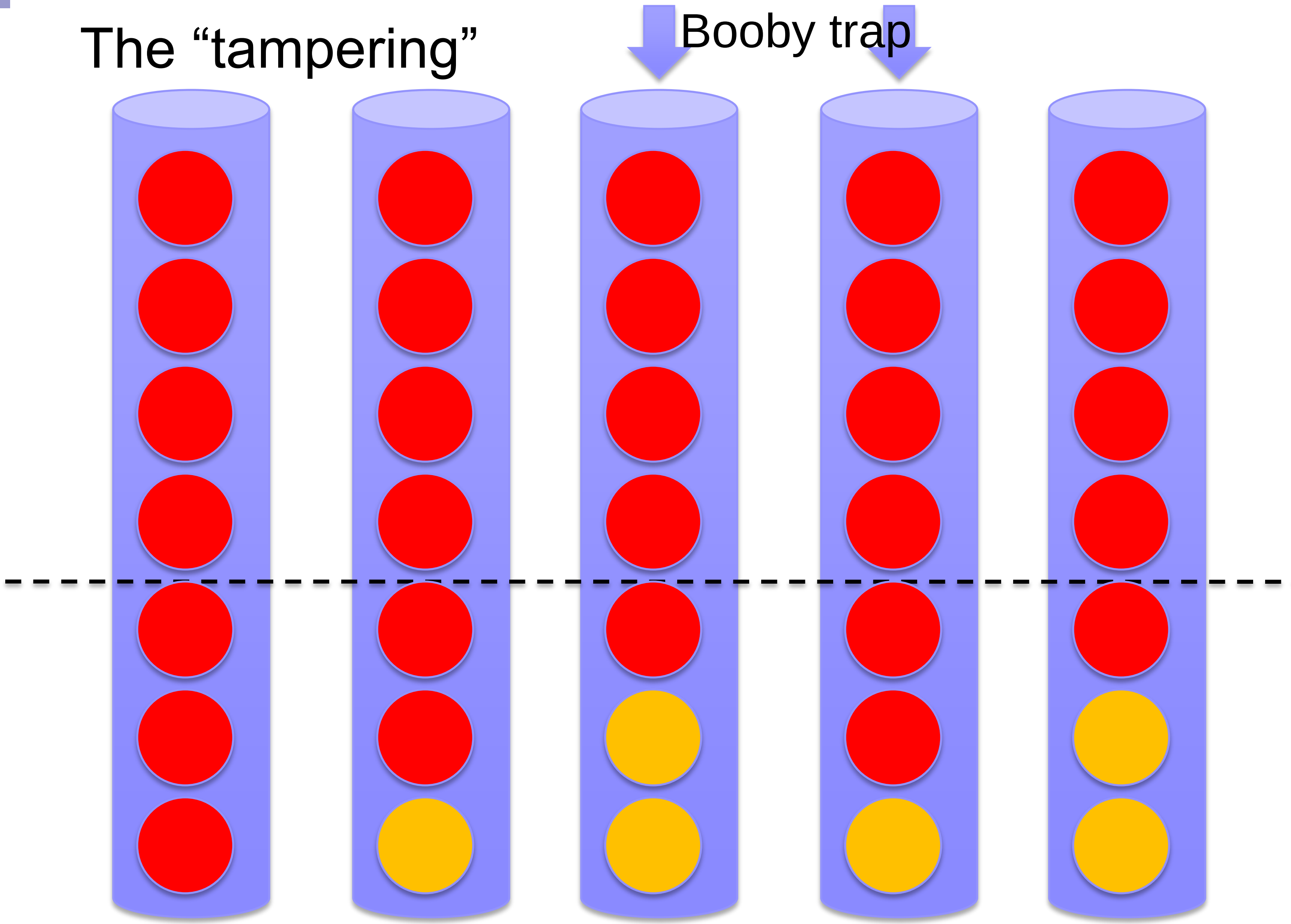
Lemon Cost

- Seller's profit on bundles from D_2 is higher by C than on bundles from D_1. (C can be >> L !)

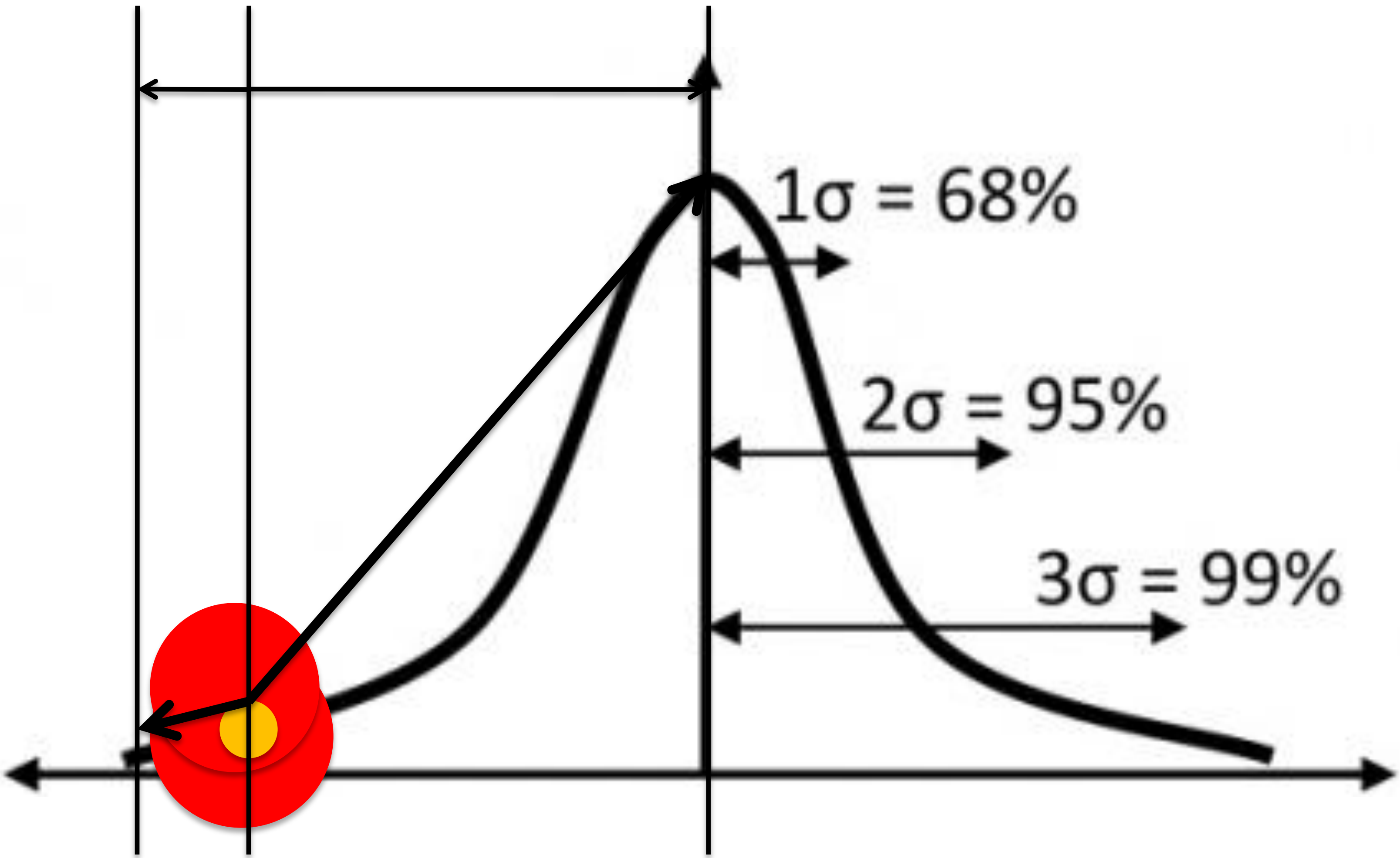
Distribution D_1



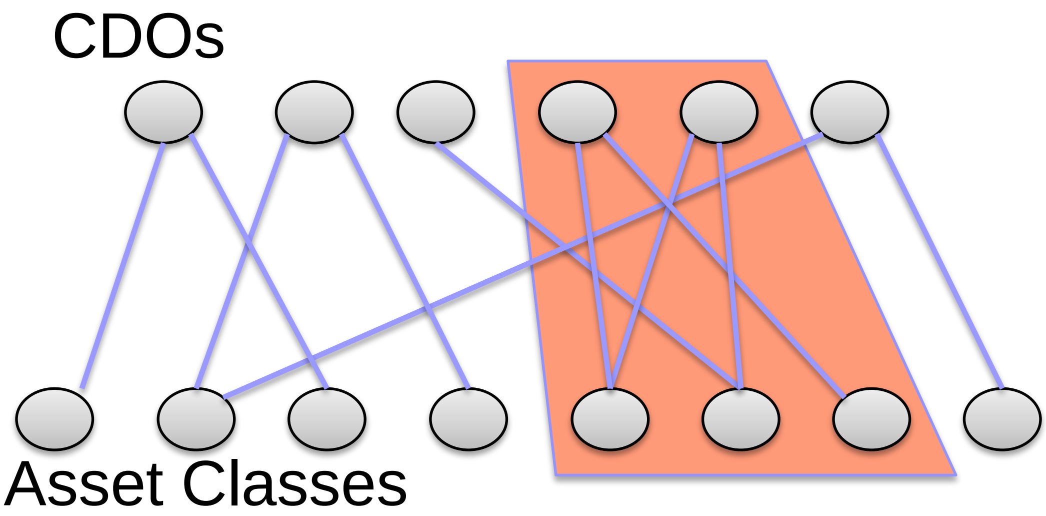
The “tampering”



Why does the seller make profit



Densest subgraph problem



Input: Graph, numbers
(k_1, k_2, e)

Output: Whether or not graph has a (k_1 x k_2) subgraph with e edges.

- Well known to be NP-complete
- Conjecture: this is hard also on randomly-generated graphs, where the dense subgraph is "planted" .
- Used in public-key cryptosystem. Applebaum et al. (2009)

Lemon costs for various derivative types

D_1: Graphs with no dense subgraph
D_2: Graphs with as large a planted subgraph as is undetectable by known algorithms

Derivative type	Fully rational buyer (D_1)	Computationally limited buyers (i.e. D_2)
Binary CDO	<< L	>> L
Tranched CDO	L/d	$L/d^{1/2}$
Binary CDO^2	→ 0	**As high as N/4**
Tranched CDO^2	→0	Remains > $L/d^{1/2}$
** asymptotic results!		

Note: (i) Distinguishes between binary CDOs vs tranched CDO; CDO vs CDO^2 (ii) Binary CDOs can amplify lemon costs

Does the tampering problem go away if we have lemon laws for derivatives?

Surprising and devastating answer: There seems to be no way for a buyer to “prove” in a court that seller cheated.

Finding a proof ex post = solving a slightly different version of the densest subgraph problem!

Also, no foreseeable way for honest seller to “prove” ex ante the nonexistence of a dense subgraph. (Believed to be intractable.)

Can we design tamper-proof derivatives (so seller can't profit from hidden info)?

- We show this is possible.
- Uses "tree-of-majorities" function; more noise-tolerant. To shift yields substantially, it becomes detectable
- Points to role for combinatorial algorithms in design and rating of securities?
- Very preliminary ---proof of concept. Requires study with respect to real-life requirements.

Open problems

- Stronger intractability results by allowing real-life complications (eg correlations, timing assumptions, etc.)?

- New security design to remove the “cost of complexity”? Must account for real-life complications.

- Prove previous goal is impossible. (Requires axiomatization of goals of securitization, and showing that securities consistent with them are tamperable. We have some results…)

- Effect of intractability and cost of complexity on the economy? Snowball effect? Implications for the current crisis?

THANK YOU

Lemon costs are hard to approximate

- For portfolio of CDO's
  - Hard to approximate for some constant
  - Reduction from Max-Independent-Set
- For portfolio of CDO2's
  - Hard to approximate to $2^{(\log n)^{1/3-\varepsilon}}$
  - Reduction from Label-Cover

The financial crisis had many causes:
regulatory failure, incorrect modeling, excessive risk-taking….

Qs. Even if we fix these issues,
is there still an issue with derivative pricing?

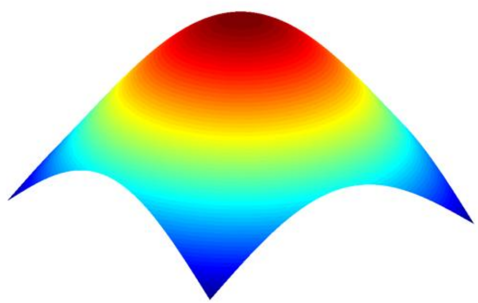
This paper: Probably yes. (Even for popular derivative types like CDO, even in popular pricing models)

- Derivative pricing is computationally intractable.
- Derivatives fail to mitigate "asymmetric info" as promised in econ. Theory
- Quantification of "complexity" of different derivatives

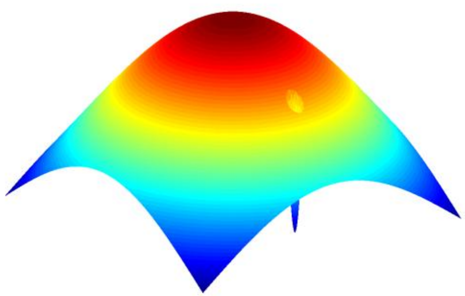
A different view of our results based upon “sensitivity”

It is possible for a fairly unsophisticated seller to design two derivatives $f_1(X_1, X_2, \ldots, X_s)$, and $f_2(X_1, X_2, \ldots, X_s)$ s.t.

- Every computationally limited actor prices them equally
- If some k<<s of X_i's are correlated then f_1, f_2 have widely different payoffs.



f_1



f_2

(Note: impossible if buyers are computationally unbounded; difference can be detected by exhaustive monte carlo simulations)

Example of derivatives

I want to buy a house, but have no money or income.

(Will default w.p 10%)

I want to get good but very safe returns.

(Safer than loaning to IBM, Wal-Mart, AT&T..)

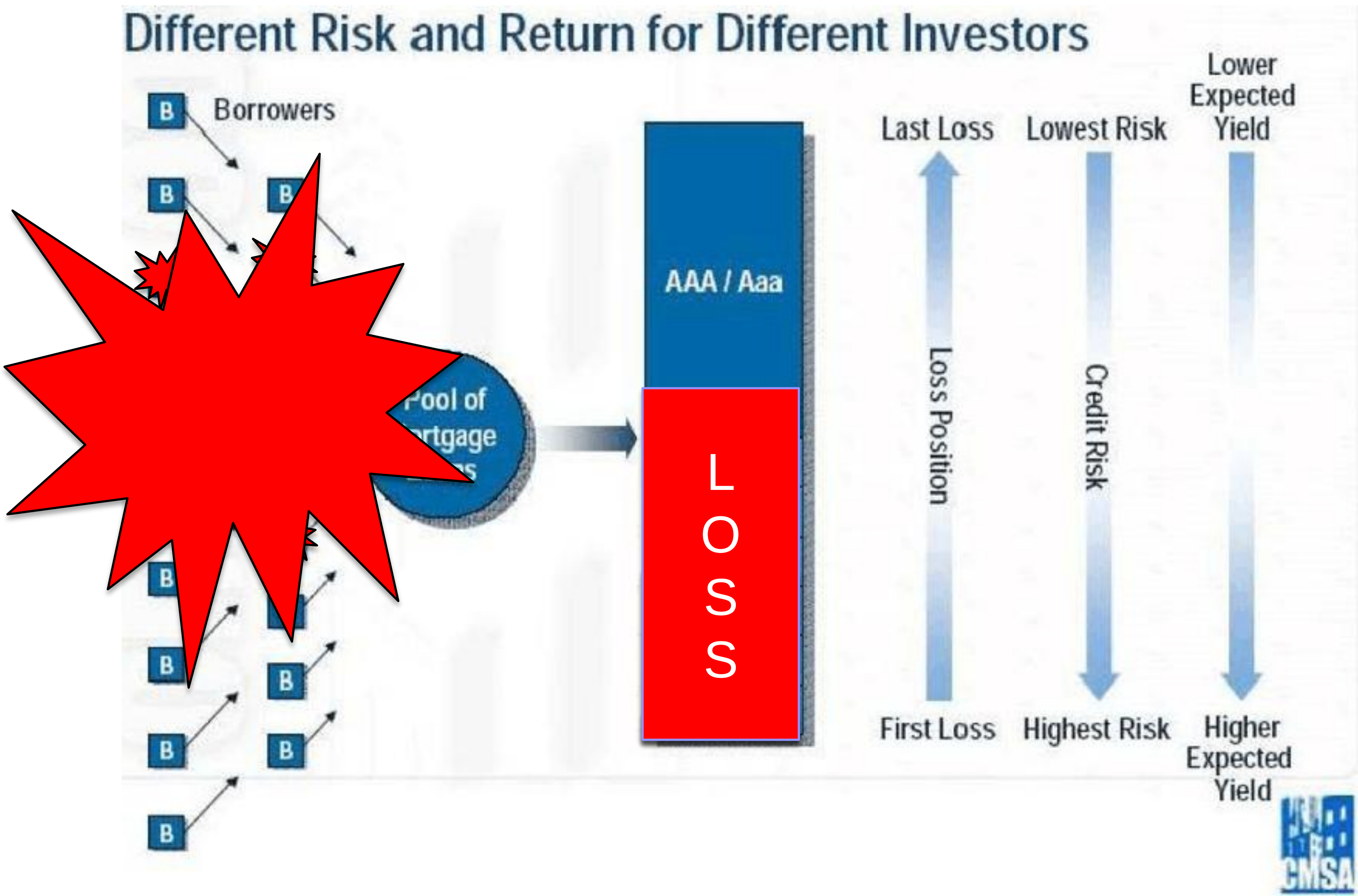
Pension fund

“Cheating by seller” does not appear to be a Nash equilibrium. Sellers must protect their reputation.

Answer 1: We only show every equilibrium in the DeMarzo type game suffers from the lemon problem. Exogeneous mechanisms like reputations (or different valuation by buyer/seller) can solve any lemon problem.

Answer 2: “I made a mistake in presuming that the self-interests of organisations, specifically banks and others, were such that they were best capable of protecting their own shareholders and their equity in the firms.” [Alan Greenspan 2008]
(describing the “flaw” in his economic philosophy)

Securitization & Tranching



Roadmap

- Derivatives (what, why etc.)
- Hiding info using complexity
  - Dense subgraph problem
- “Lemon cost due to complexity” for various derivatives
- Concluding remarks

Crash of ‘08 (in 1 slide)

US Subprime Morgages:
6.8% in value
Caused 43% foreclosures

Lesson Learnt
Derivatives have large influence.
Derivatives are “complex” and difficult to price.