Cryptographic Complexity & Computational Intractability

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The University of Montana

Crypto Means & Goals

One-Way Functions

One-Way Permutations

Trapdoor One-Way Permutations

OT protocol

Collision-Resistant Hash Functions

Zero-Knowledge Proofs

Encryption

Signatures

Homomorphic Encryption

Secret Communication Channels

Authenticated Communication Channels

OT Channel

Digital Cash

Mental Poker

Privacy Preserving Data-Mining

E-Voting

Intractability

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- Reductions represent cryptographic goals (cf. algorithmic goals)

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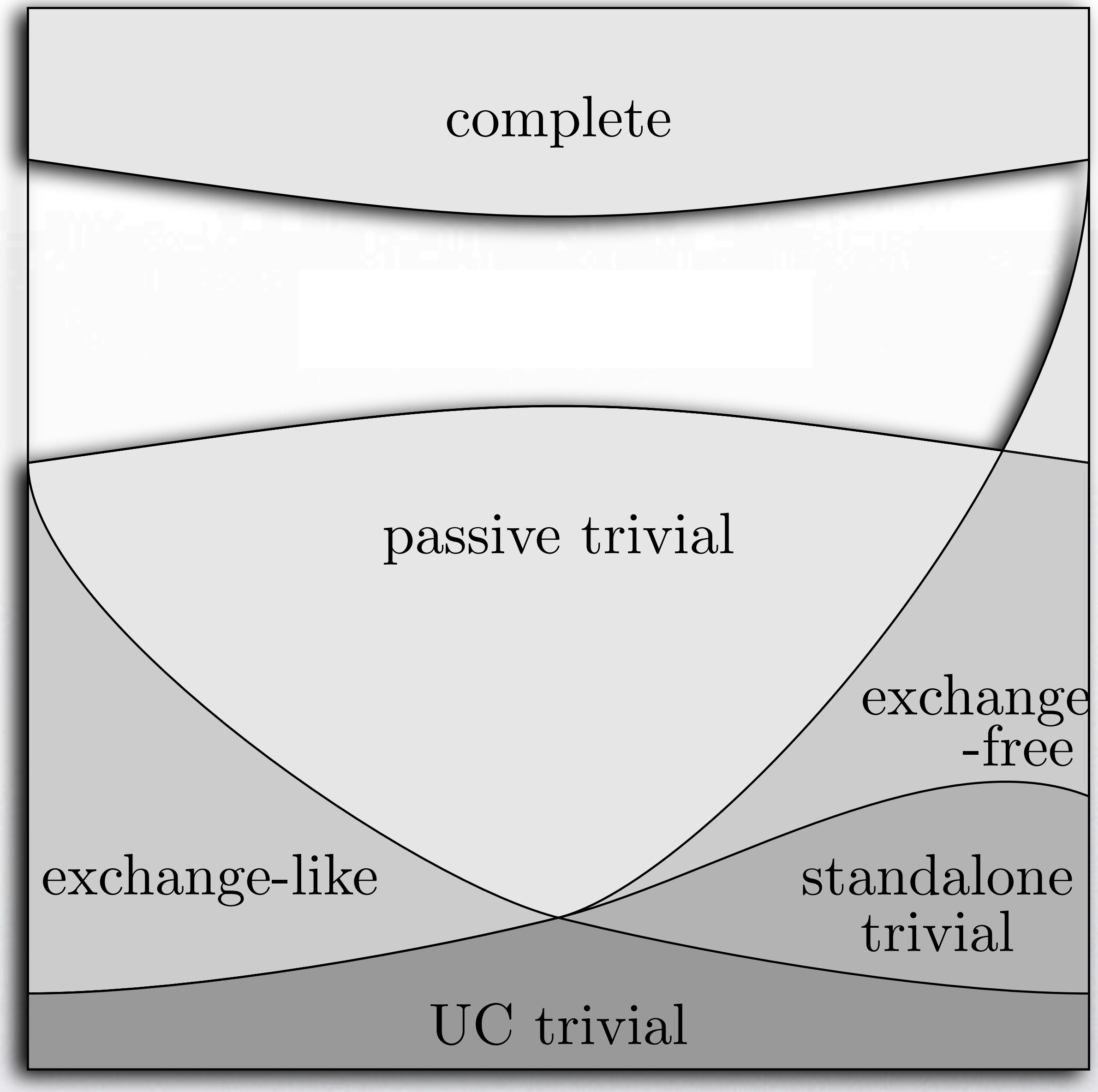

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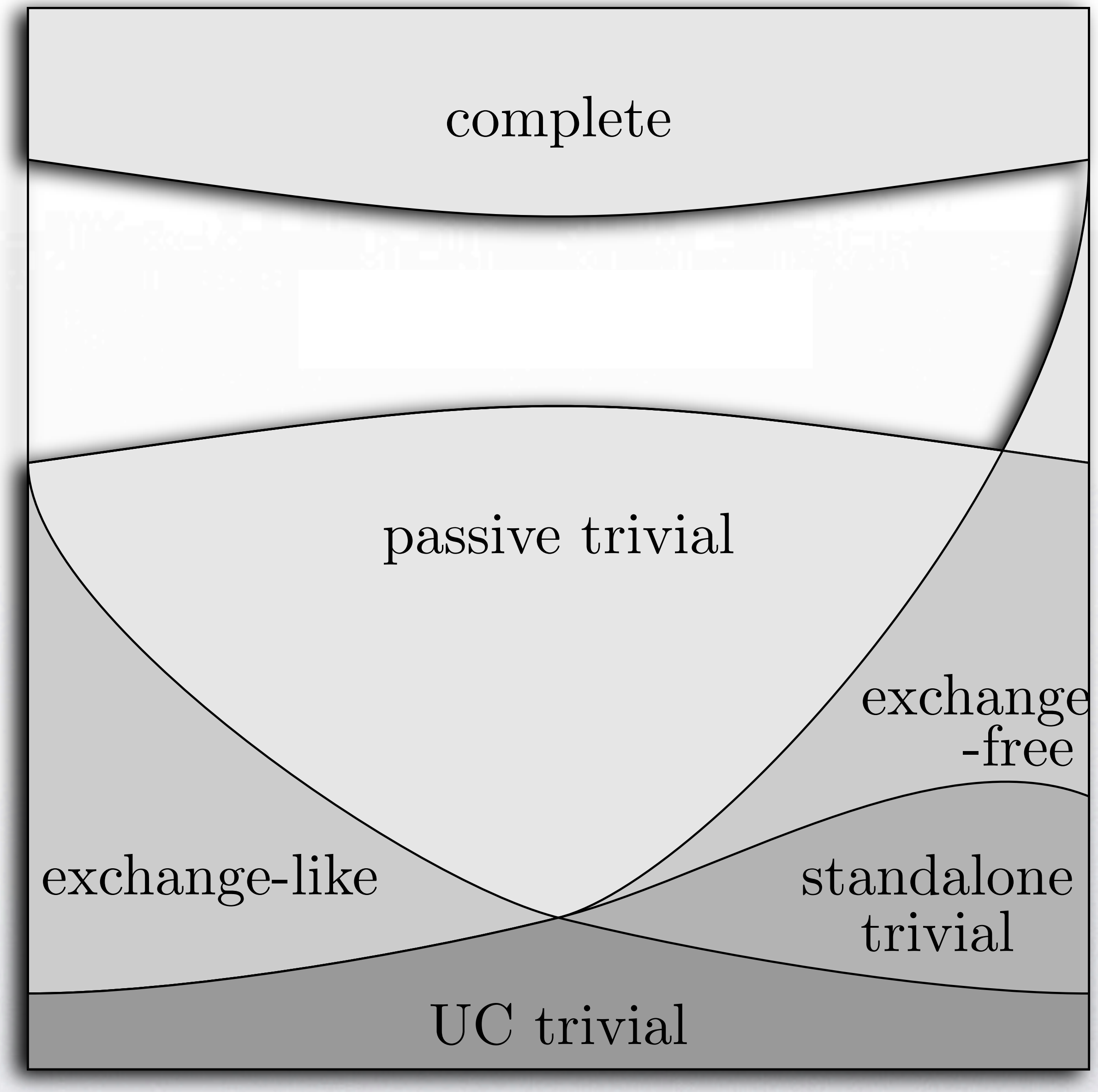

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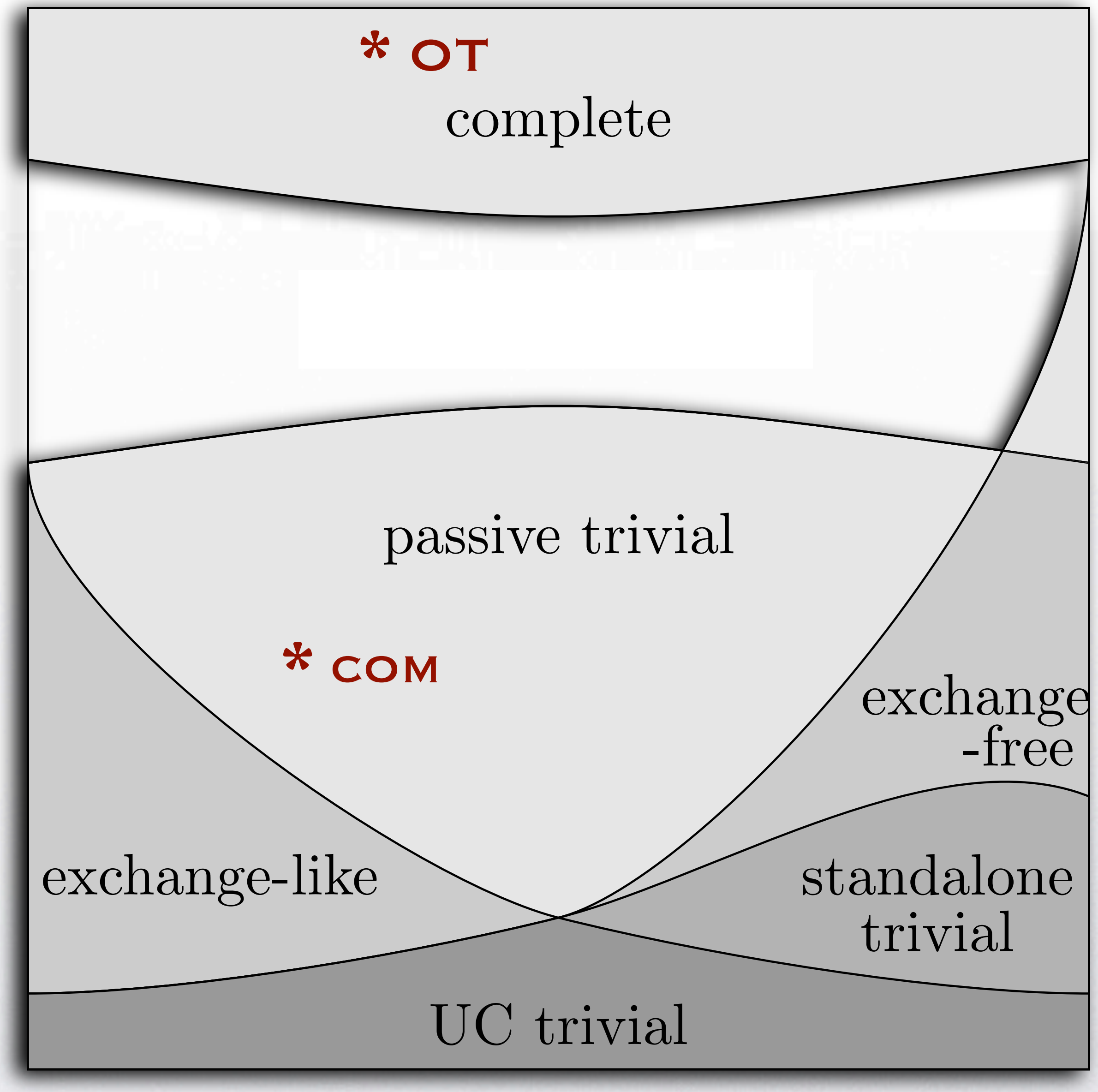

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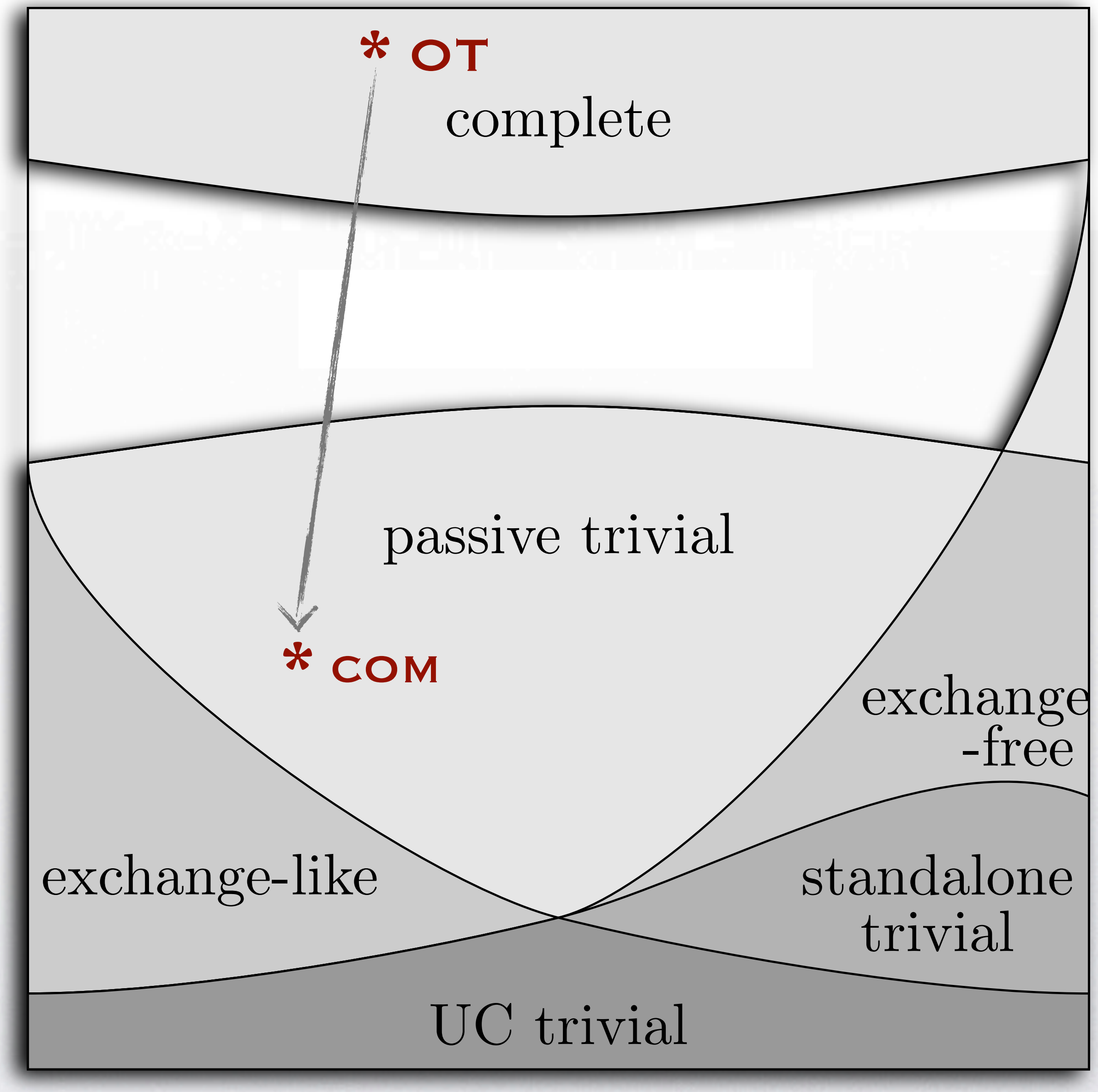

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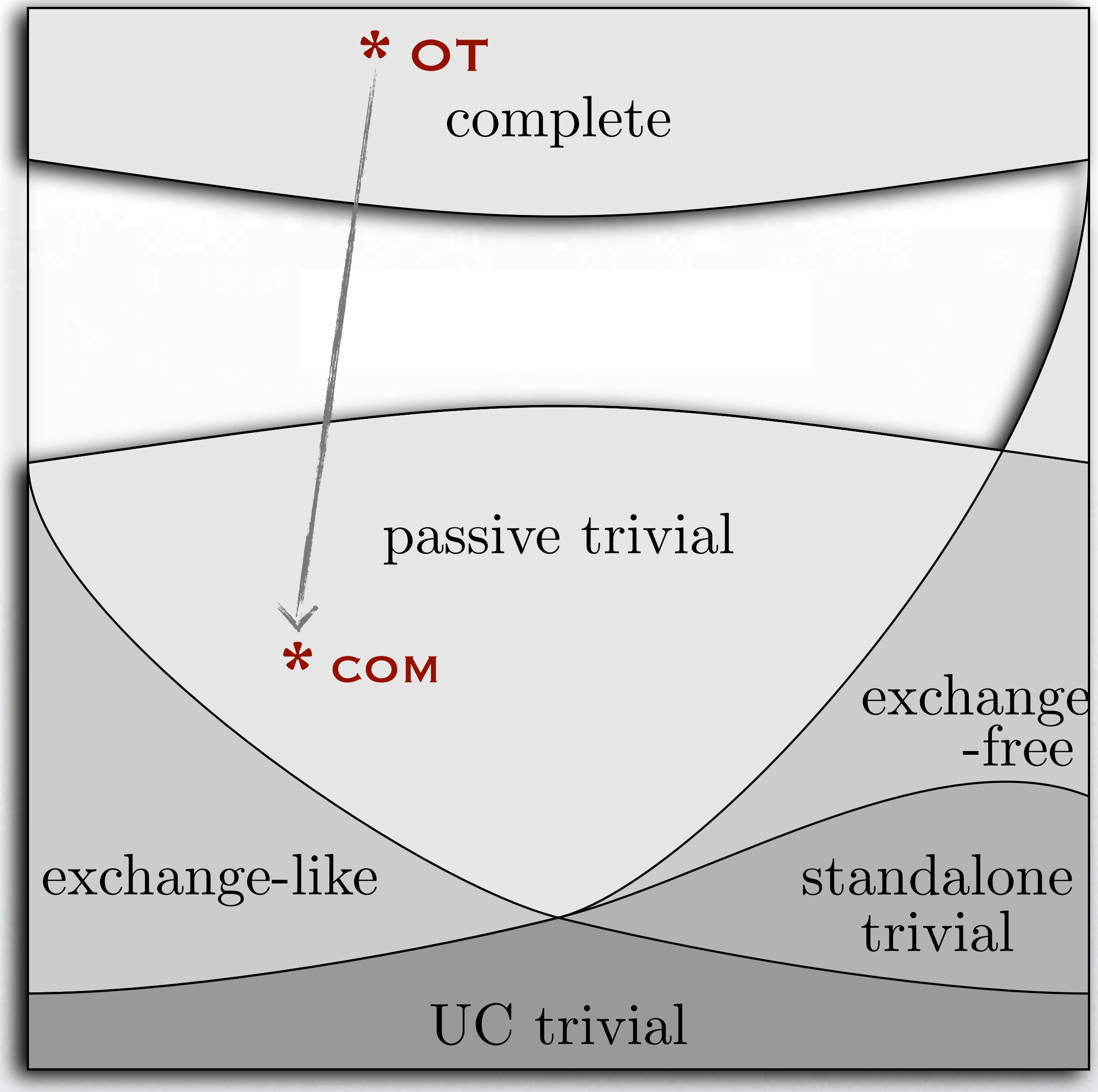

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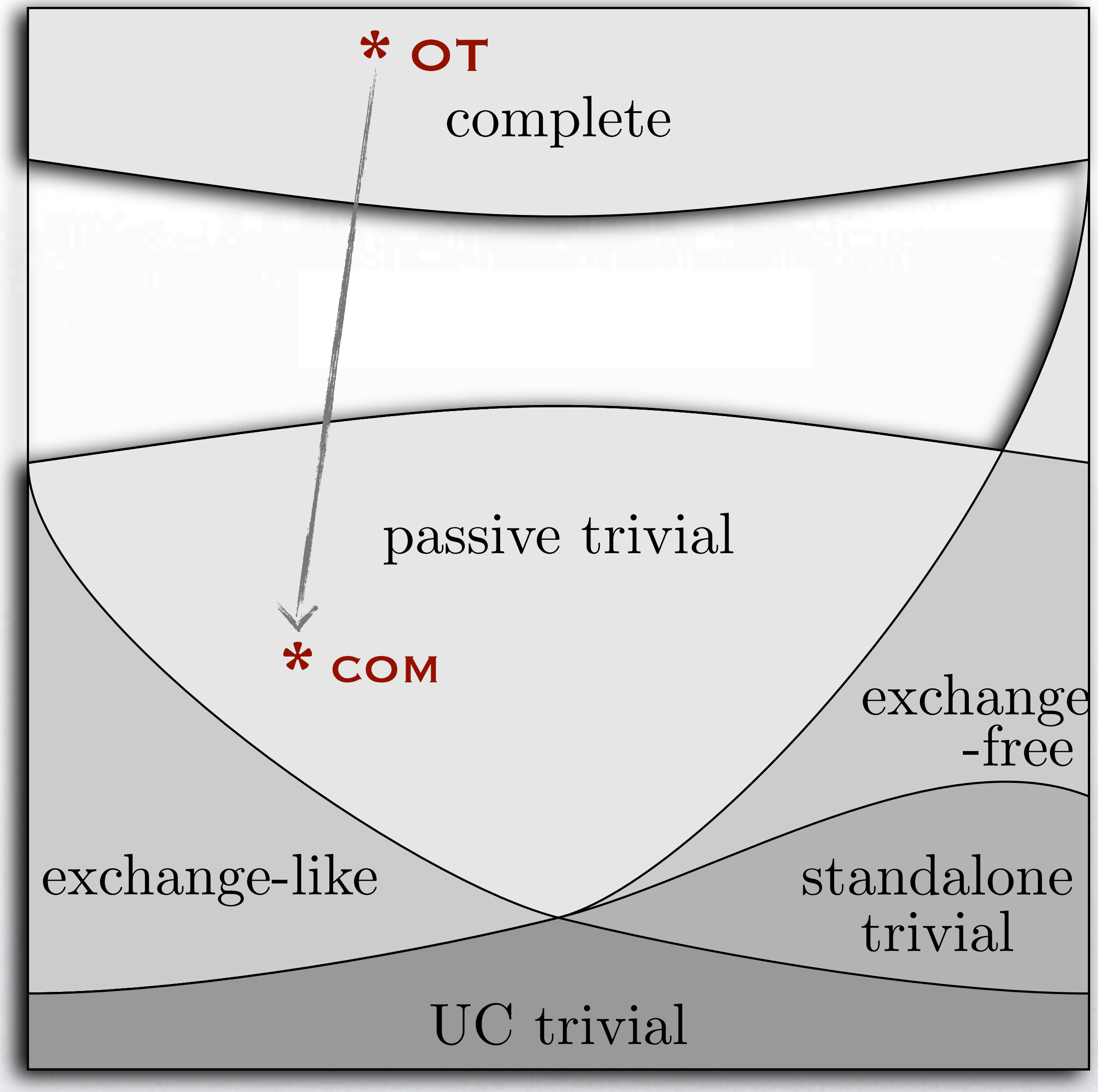

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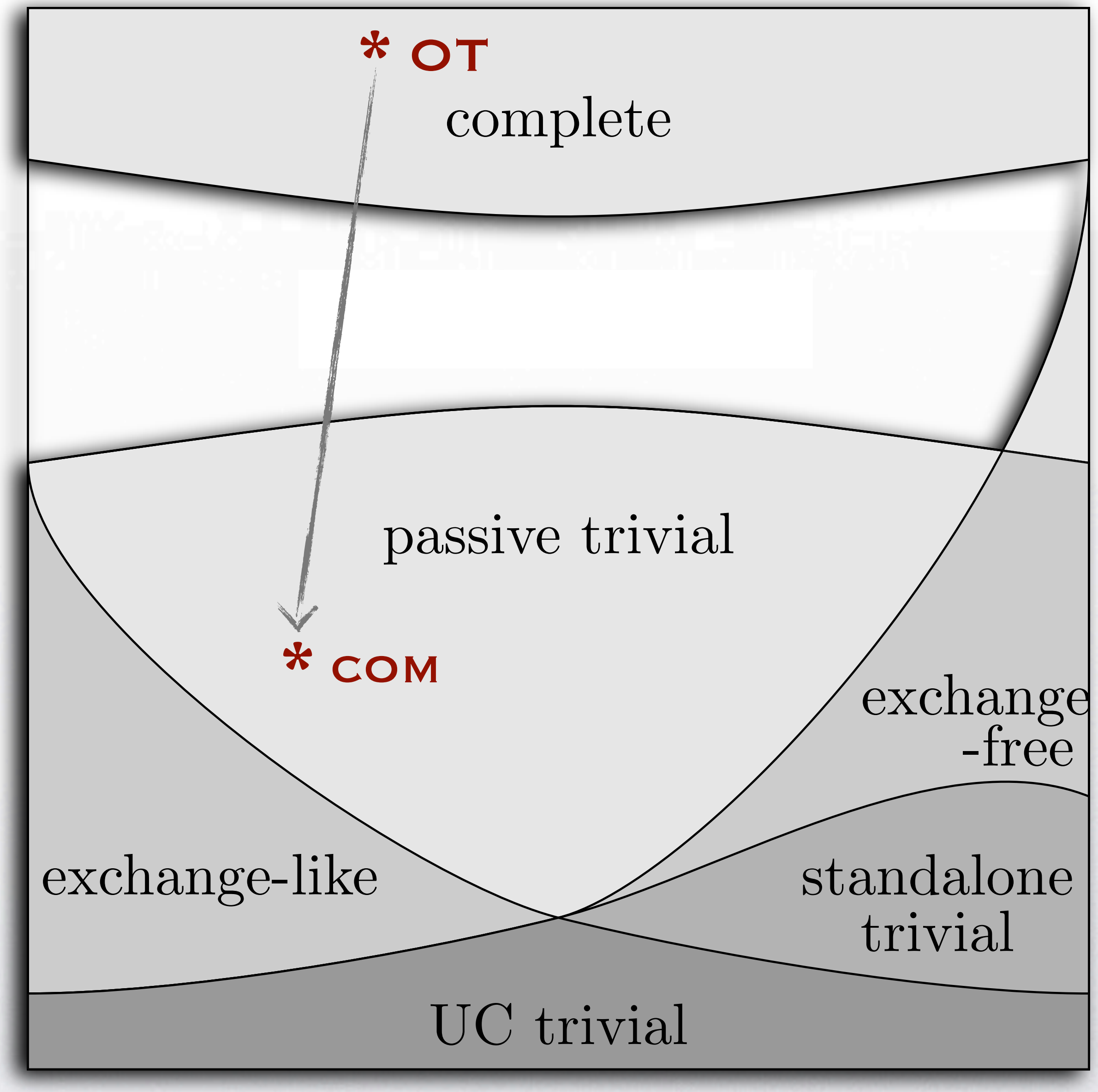

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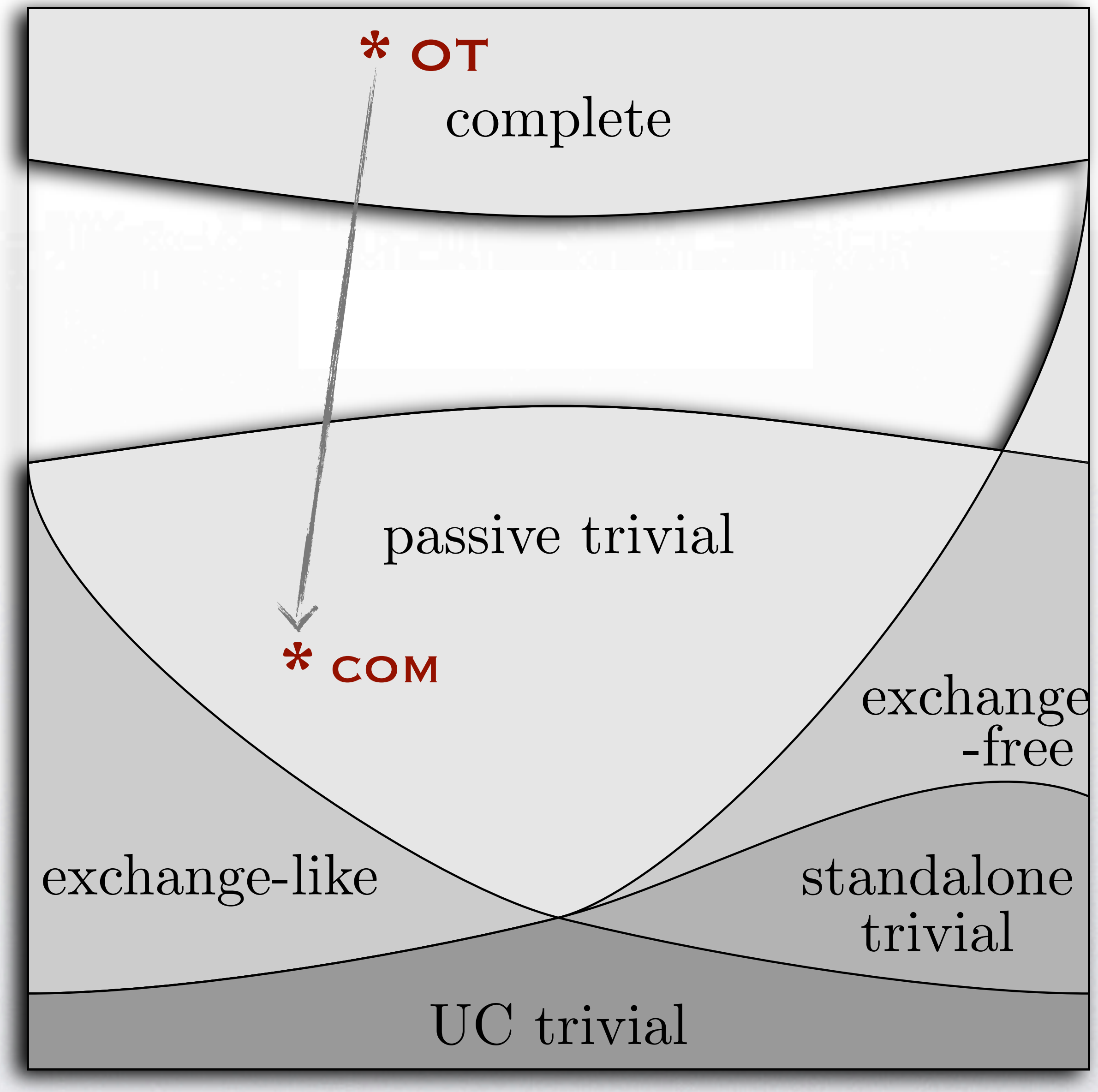

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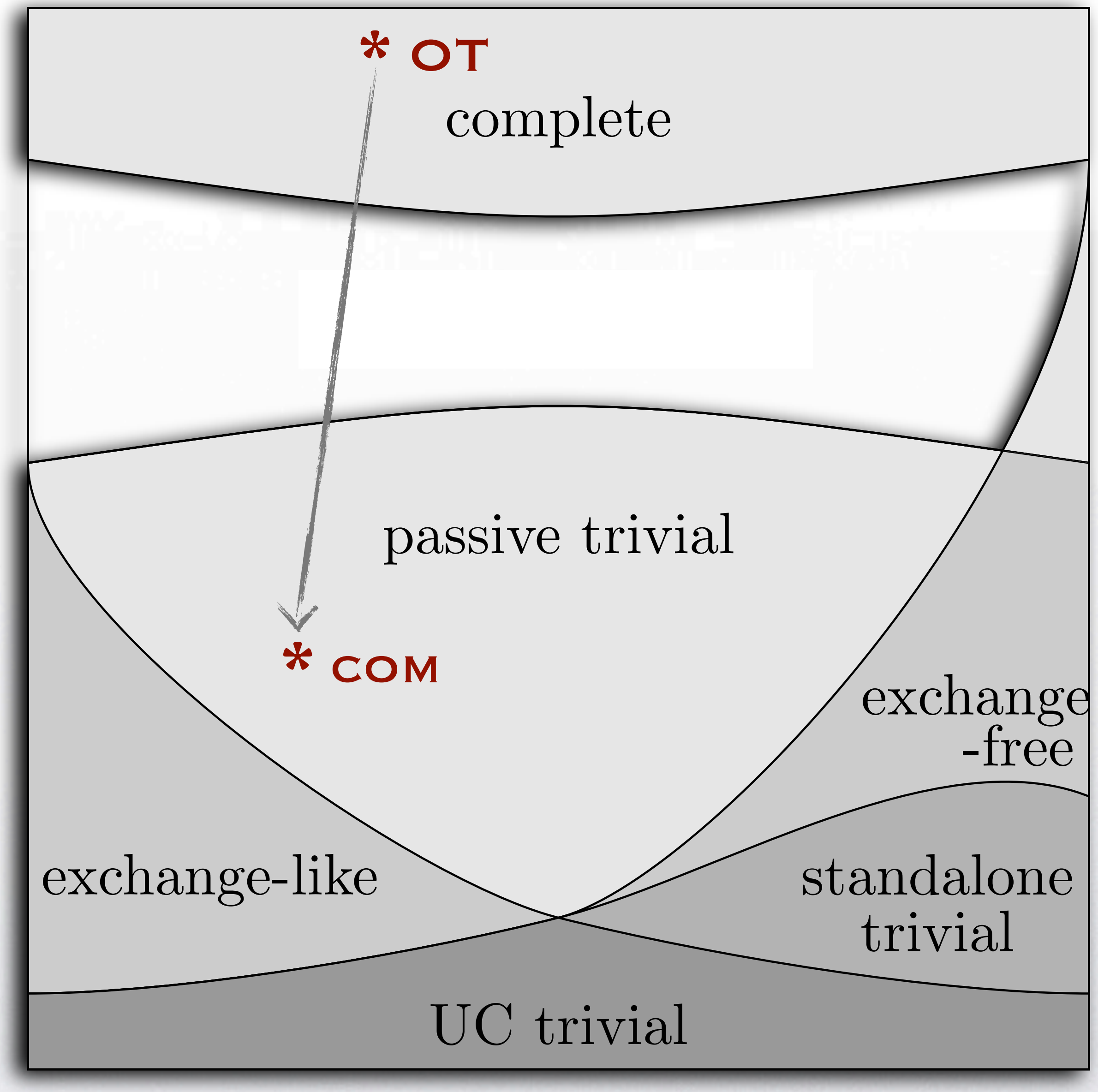

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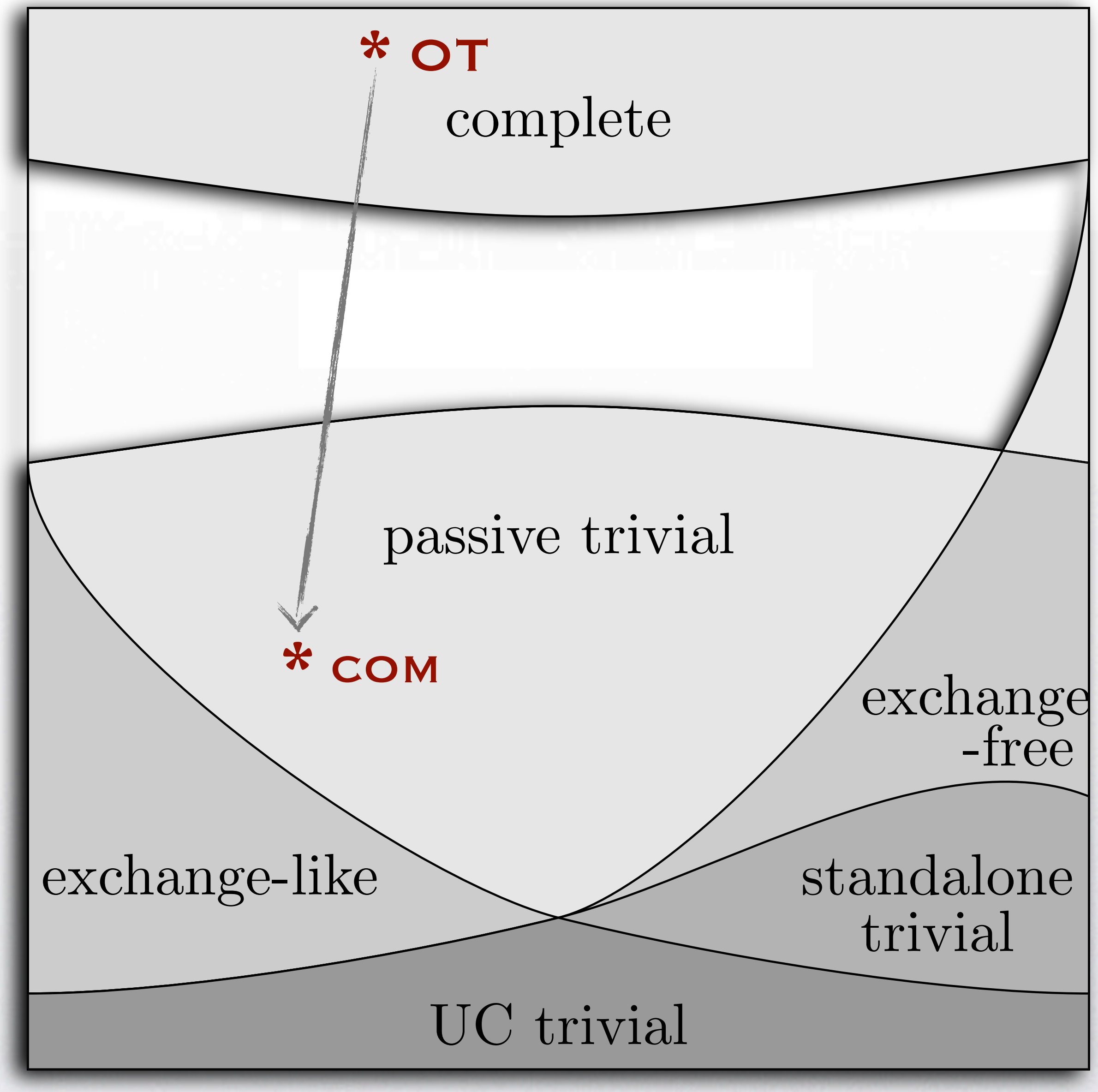
- Computationally unbounded setting



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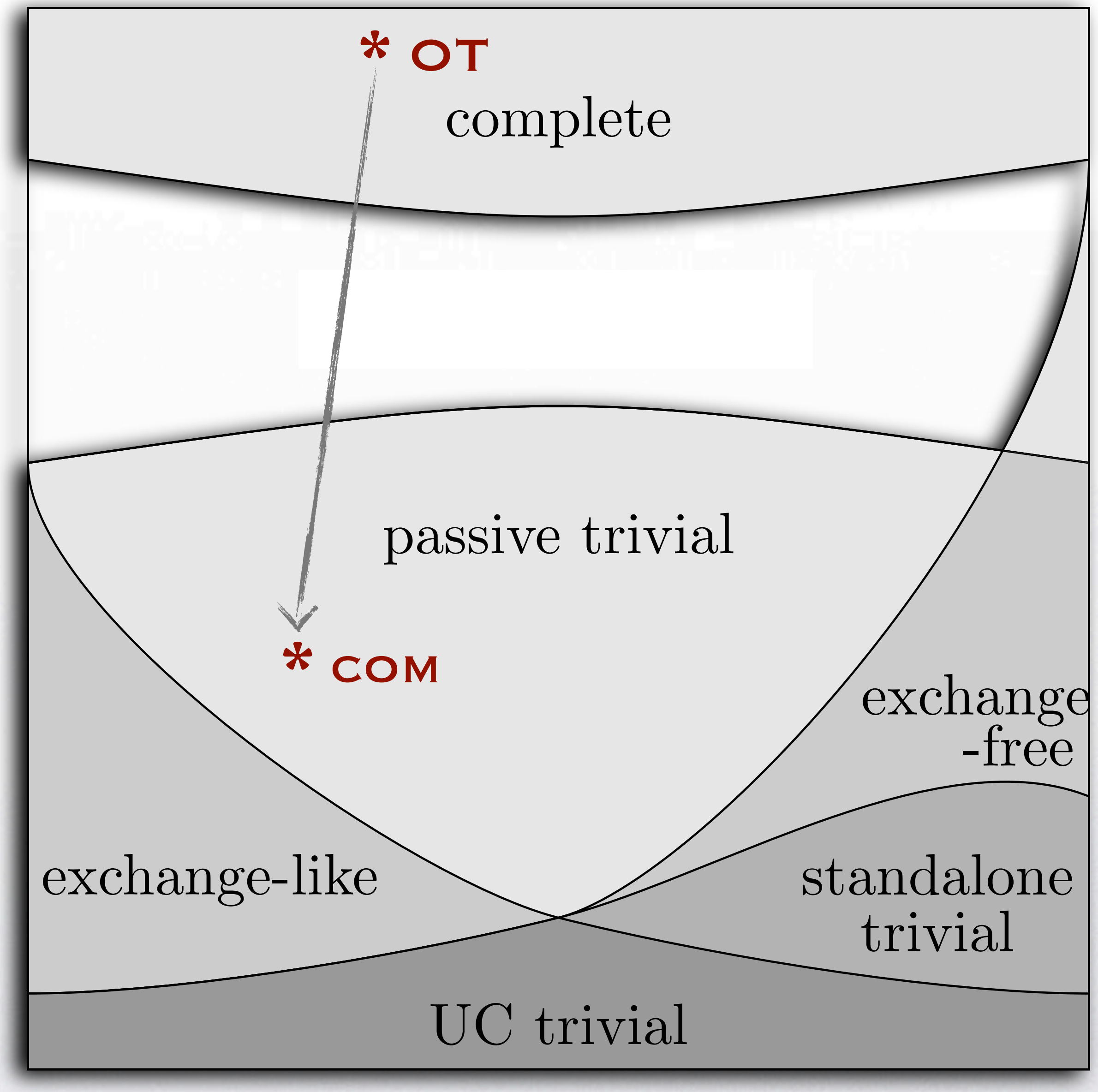
  - Universe of assumptions: $F \sqsubseteq G$ in the computationally bounded setting

Assumptions: F ⊑ G



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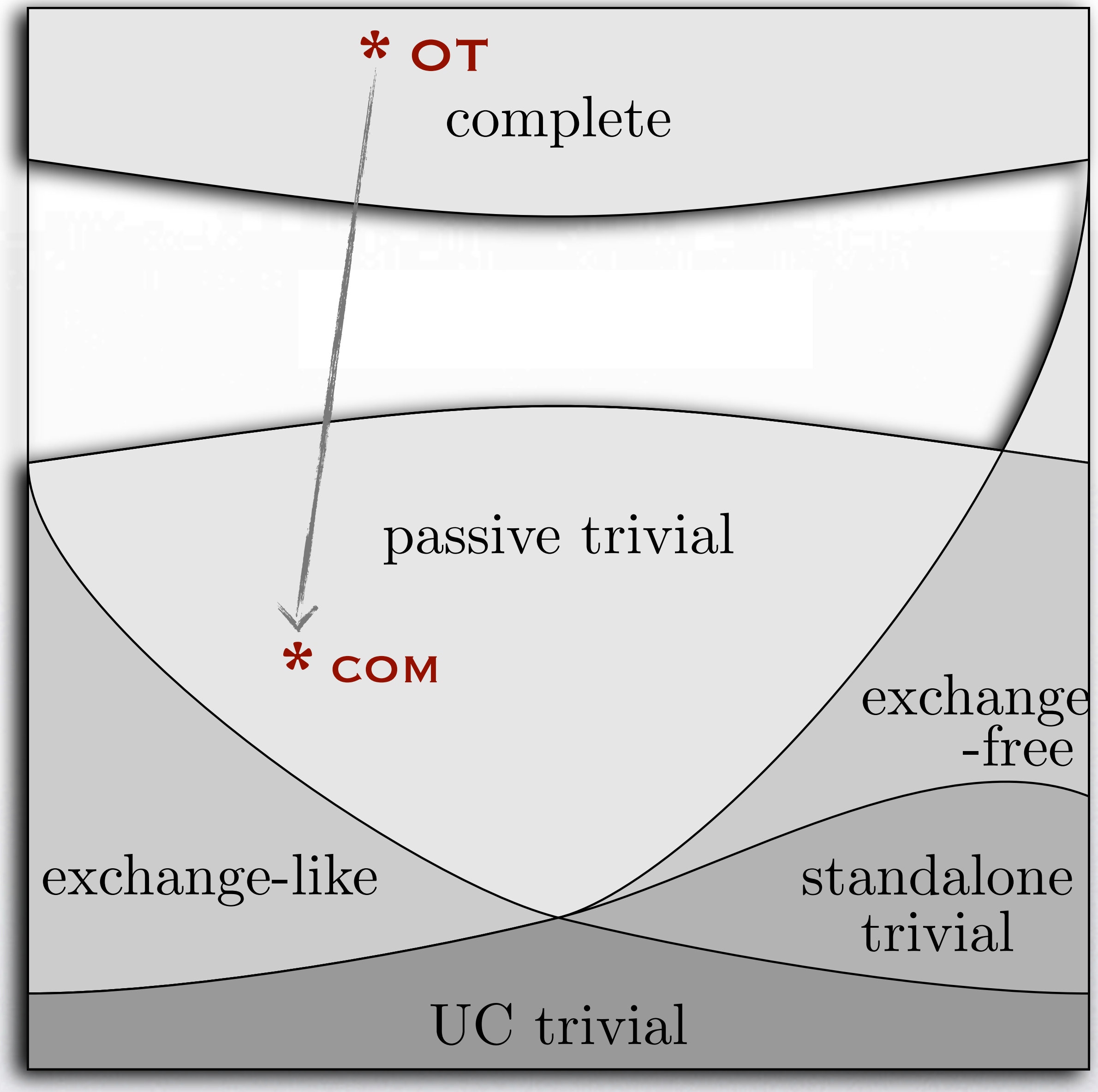

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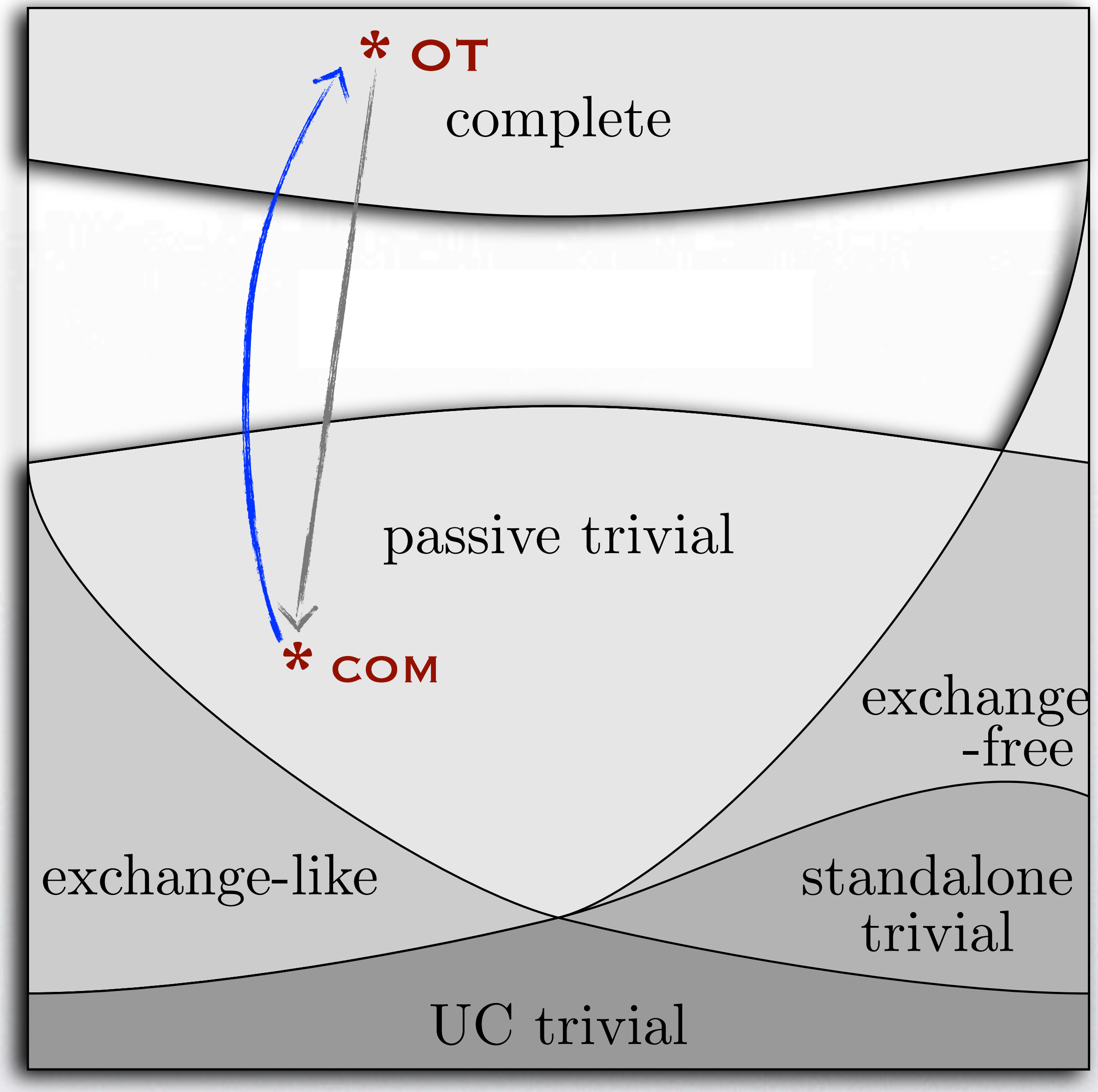

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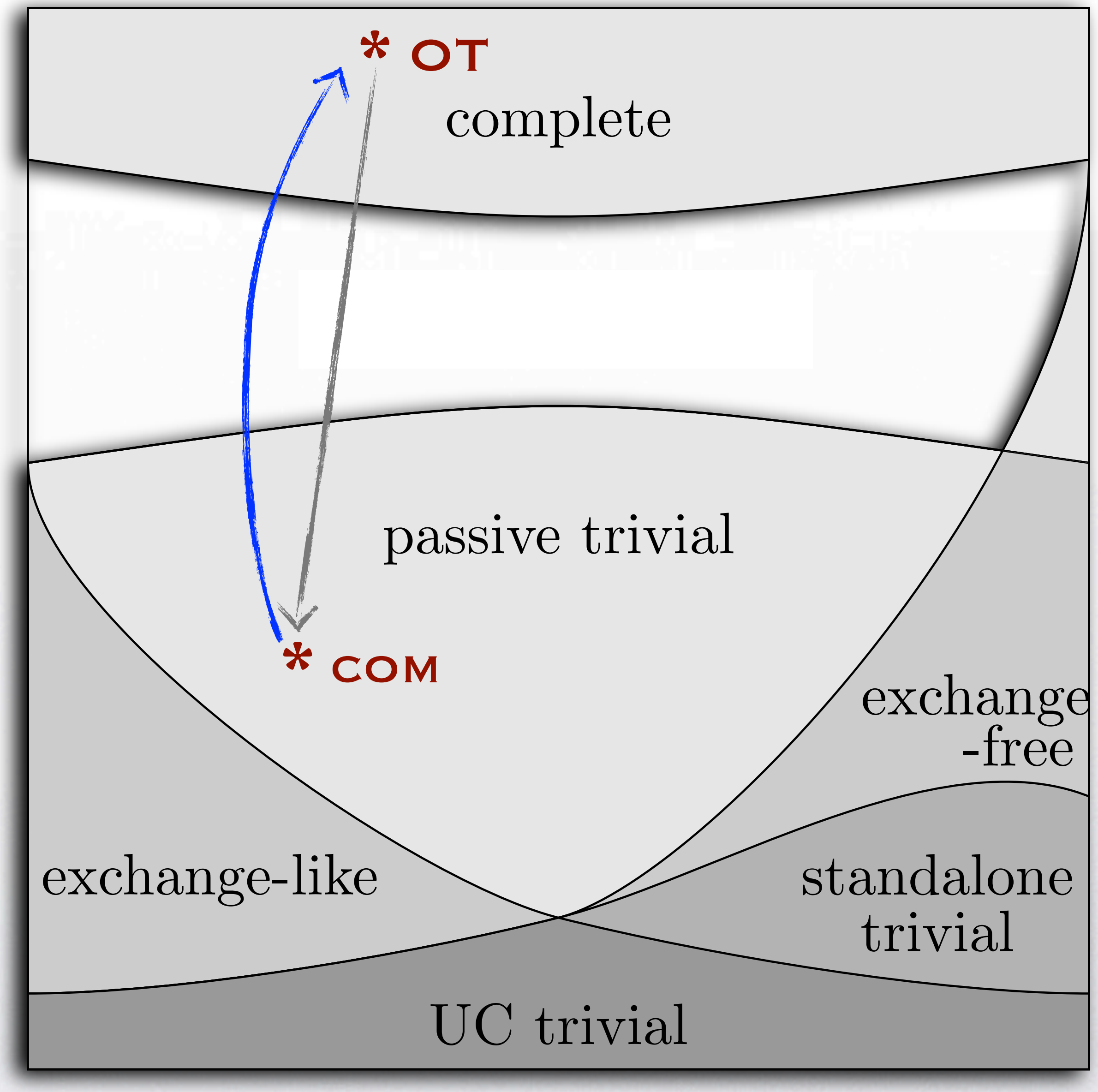

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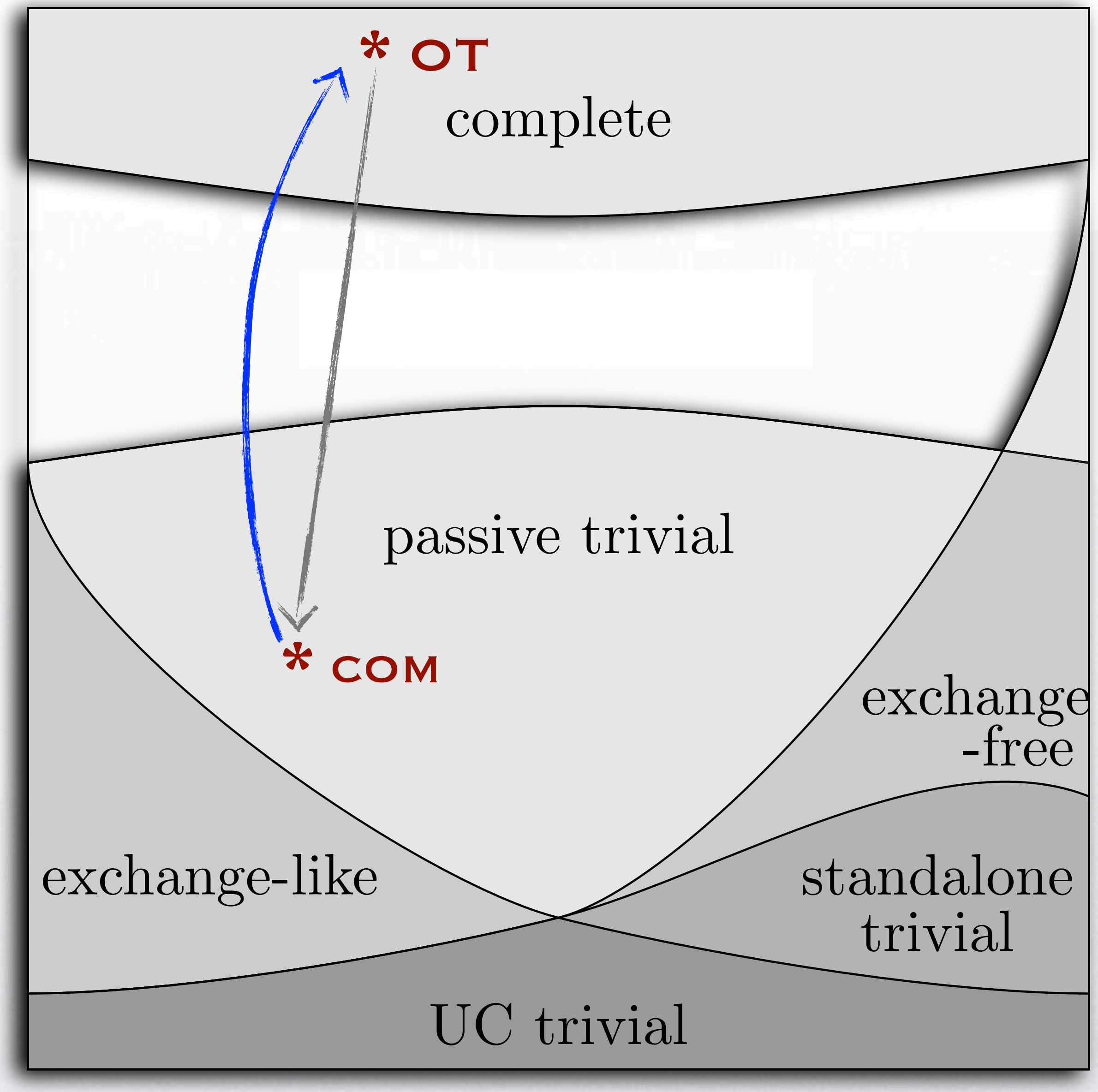
- Can consider multiple notions of ⊑. Here, UC security against active (static) adversaries.



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  - Many standard general assumptions (like OWP) may not appear in our universe

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      - In particular shOT is *the maximal assumption*

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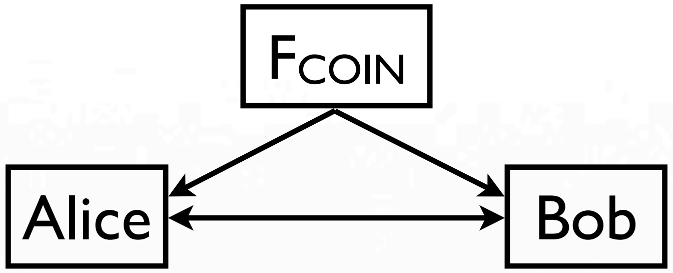

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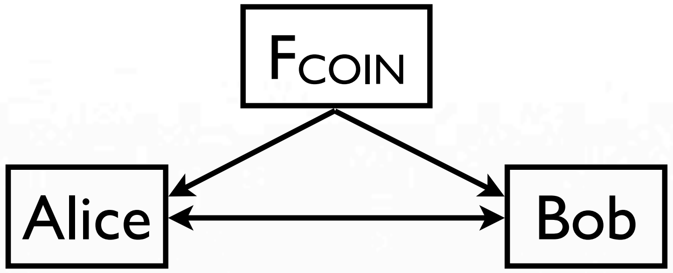

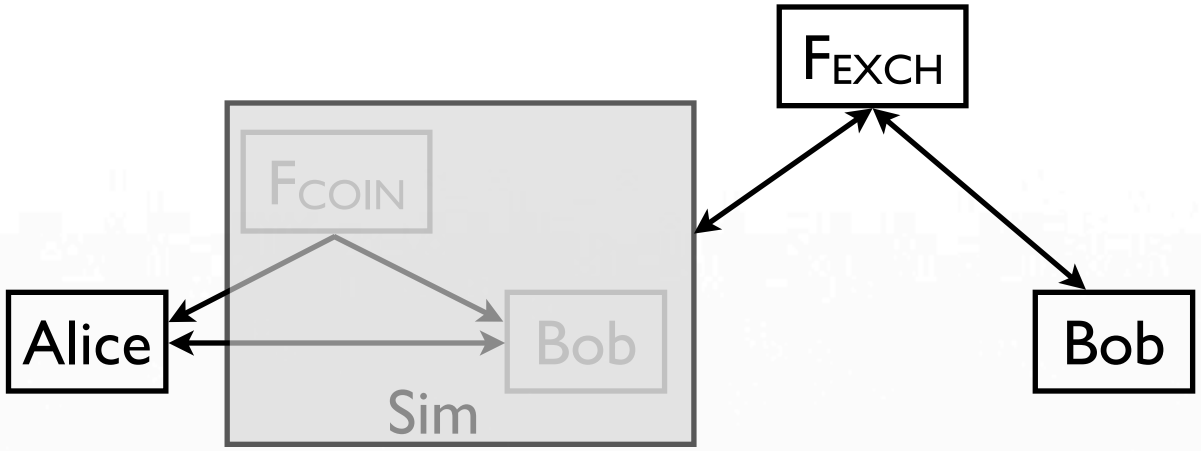

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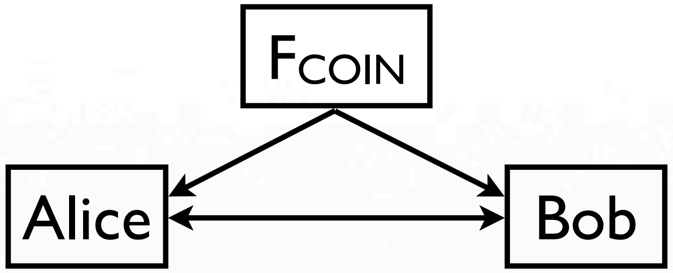

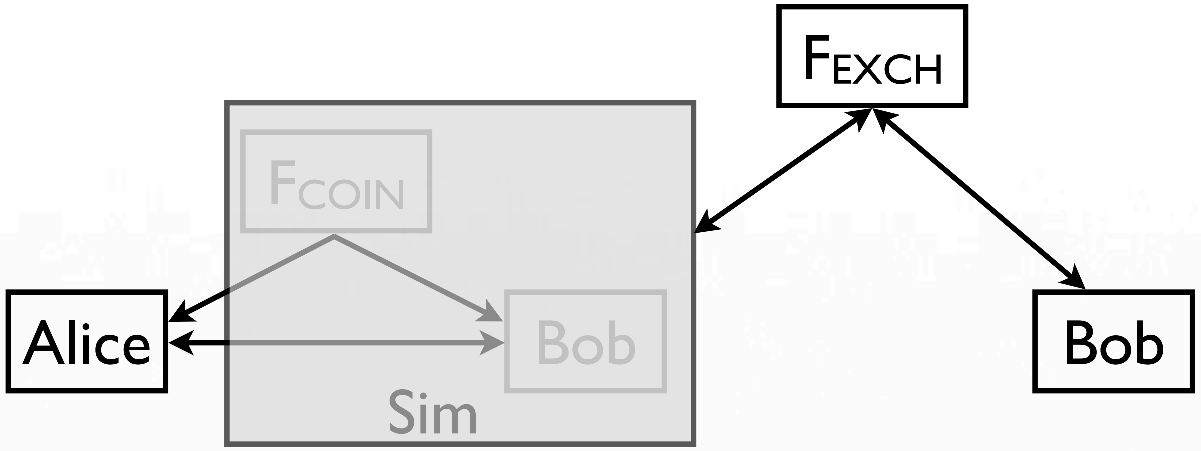

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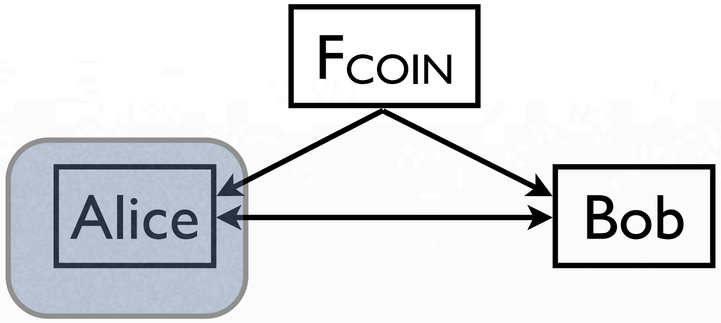

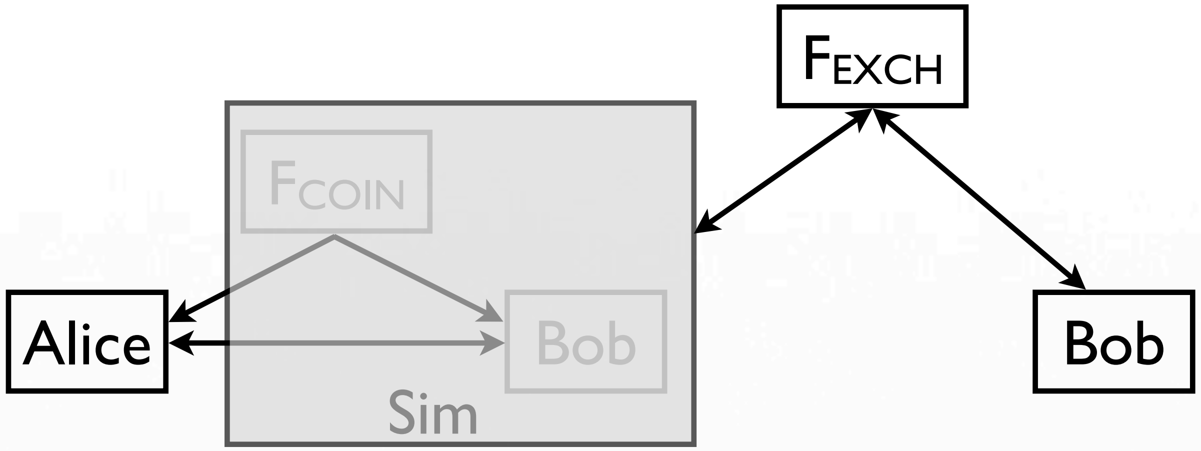

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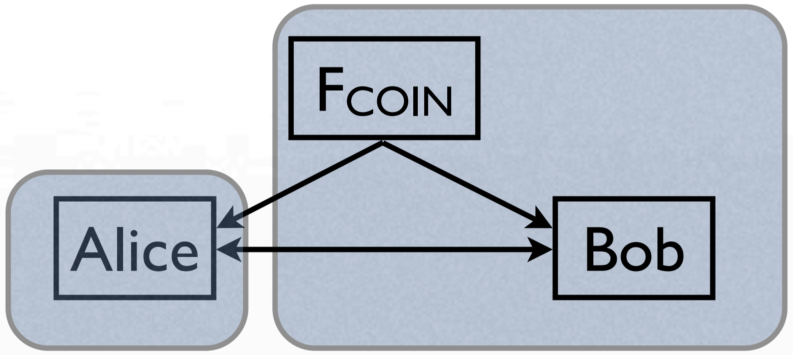

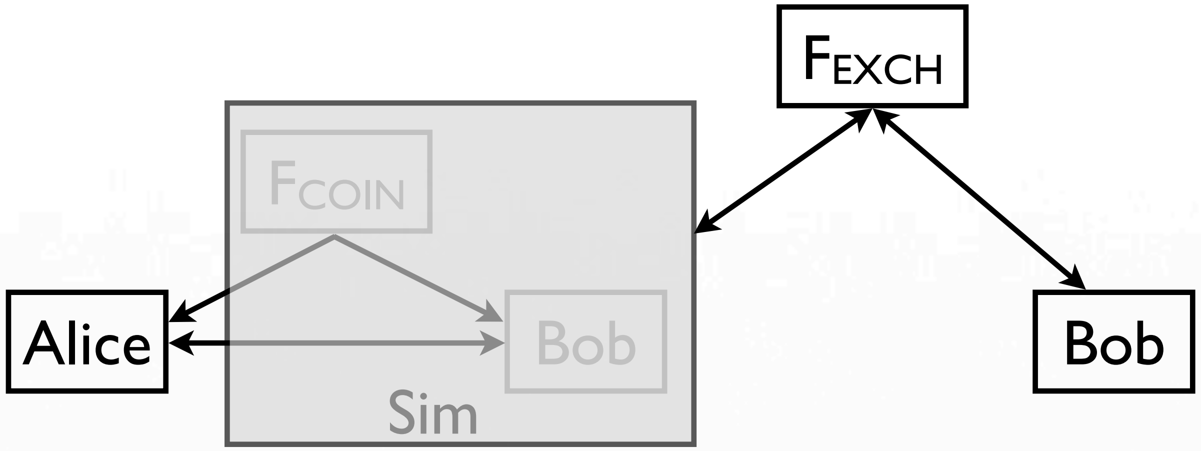

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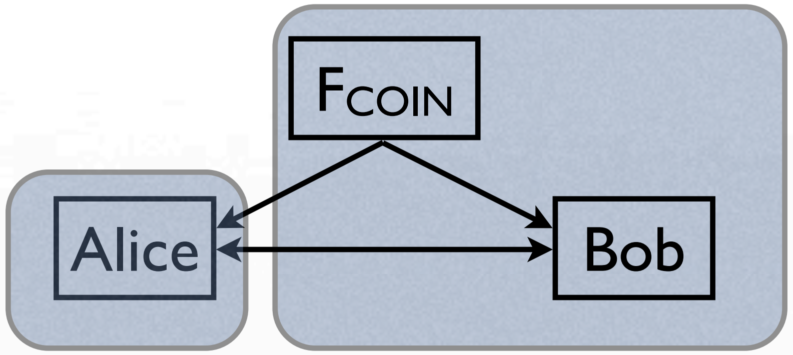

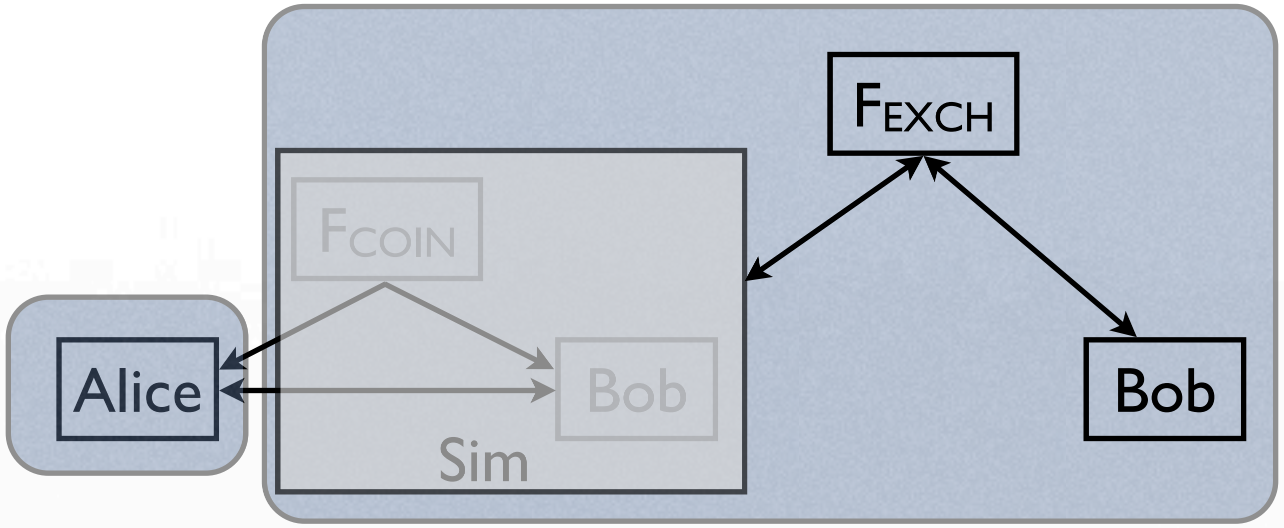

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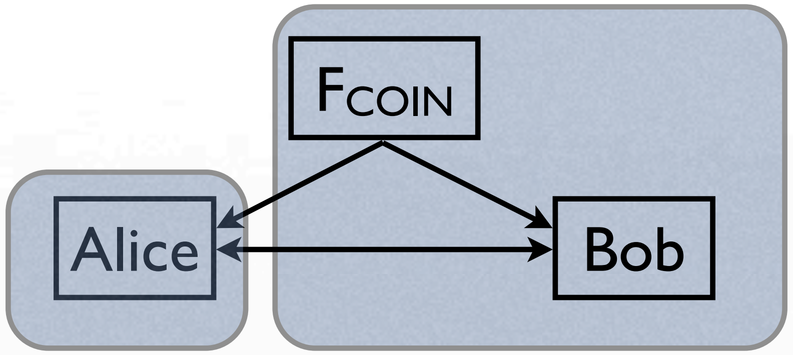

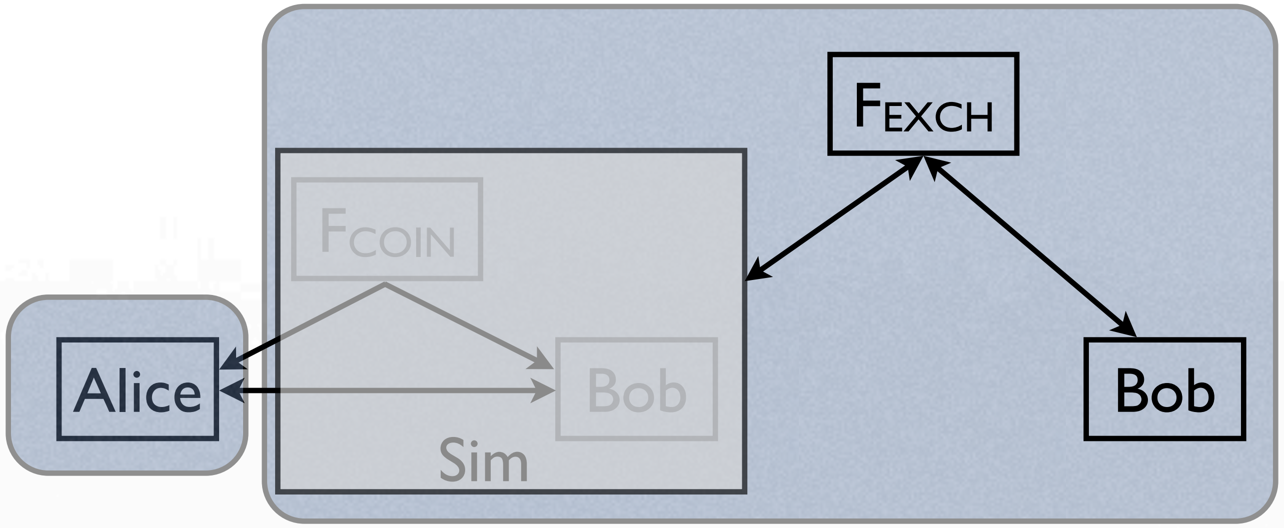

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  - Truncate the execution at a random round

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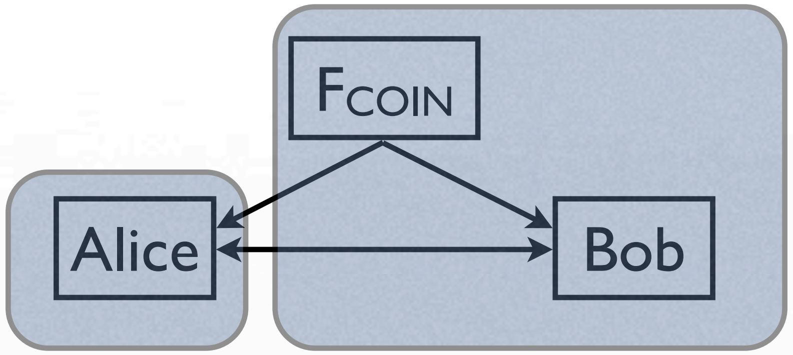

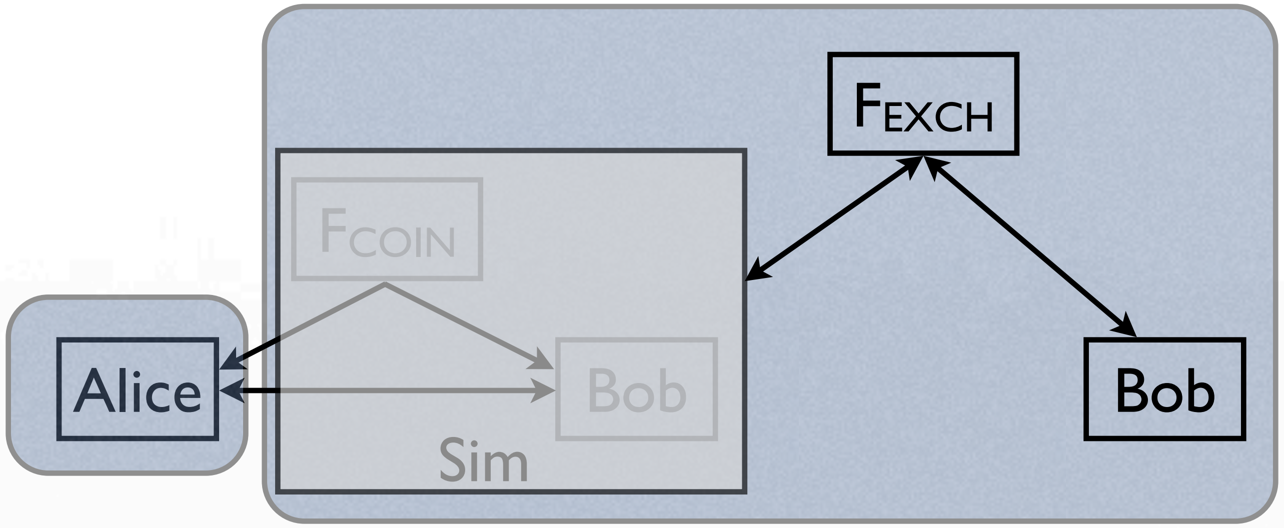

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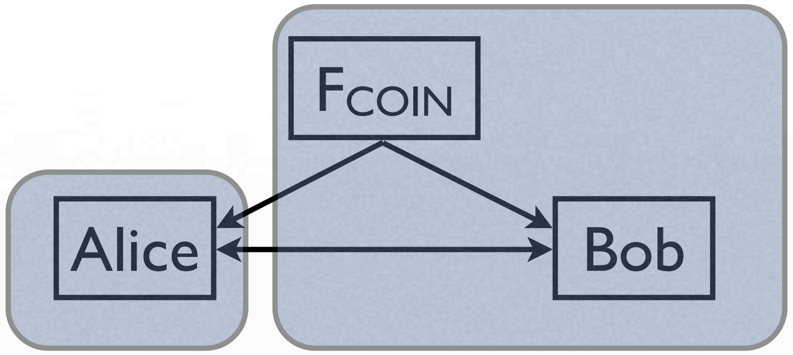

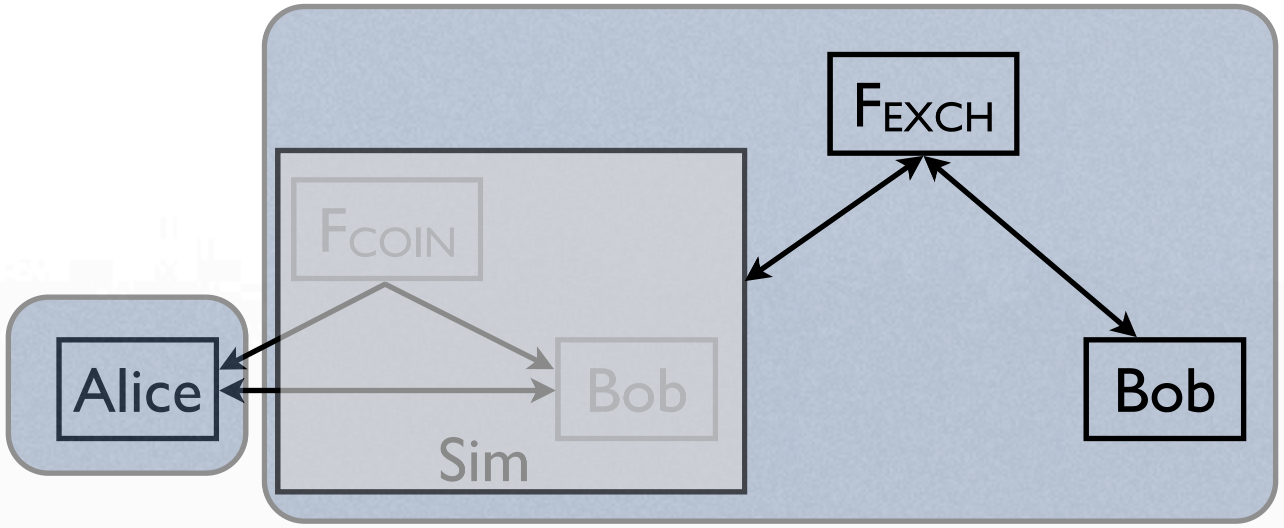

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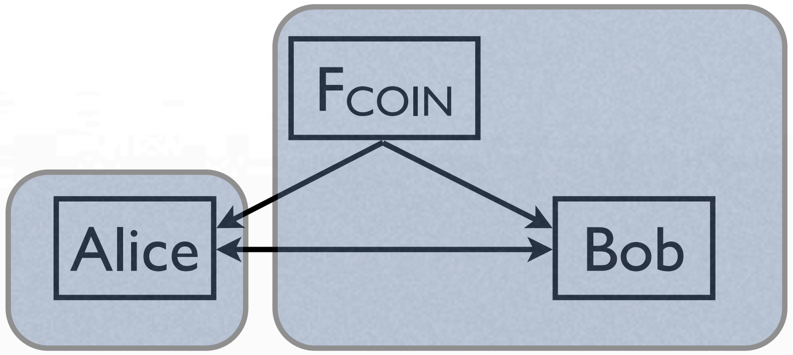

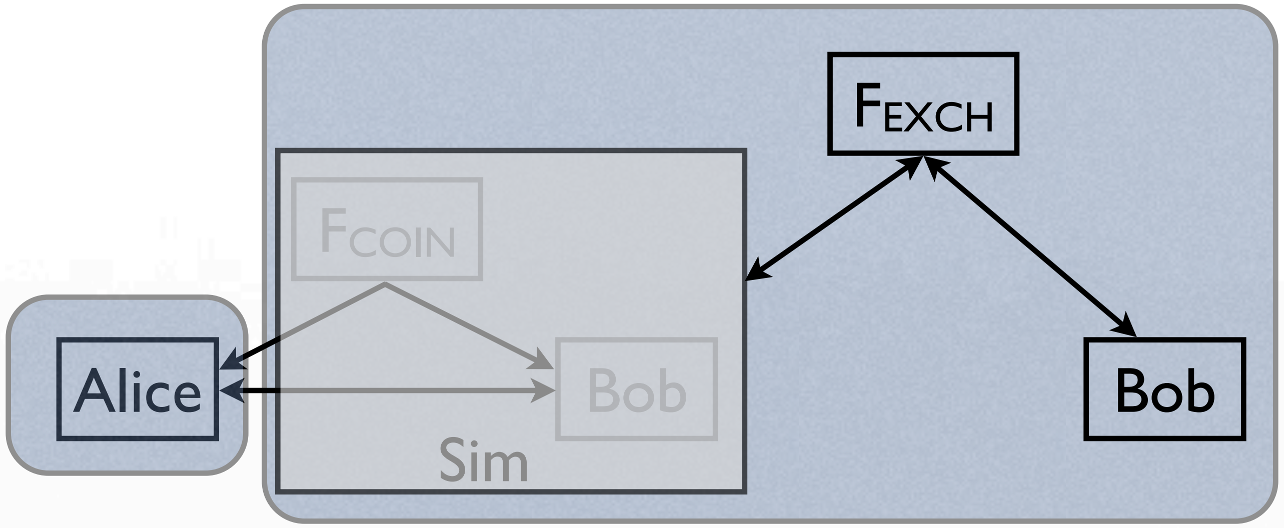

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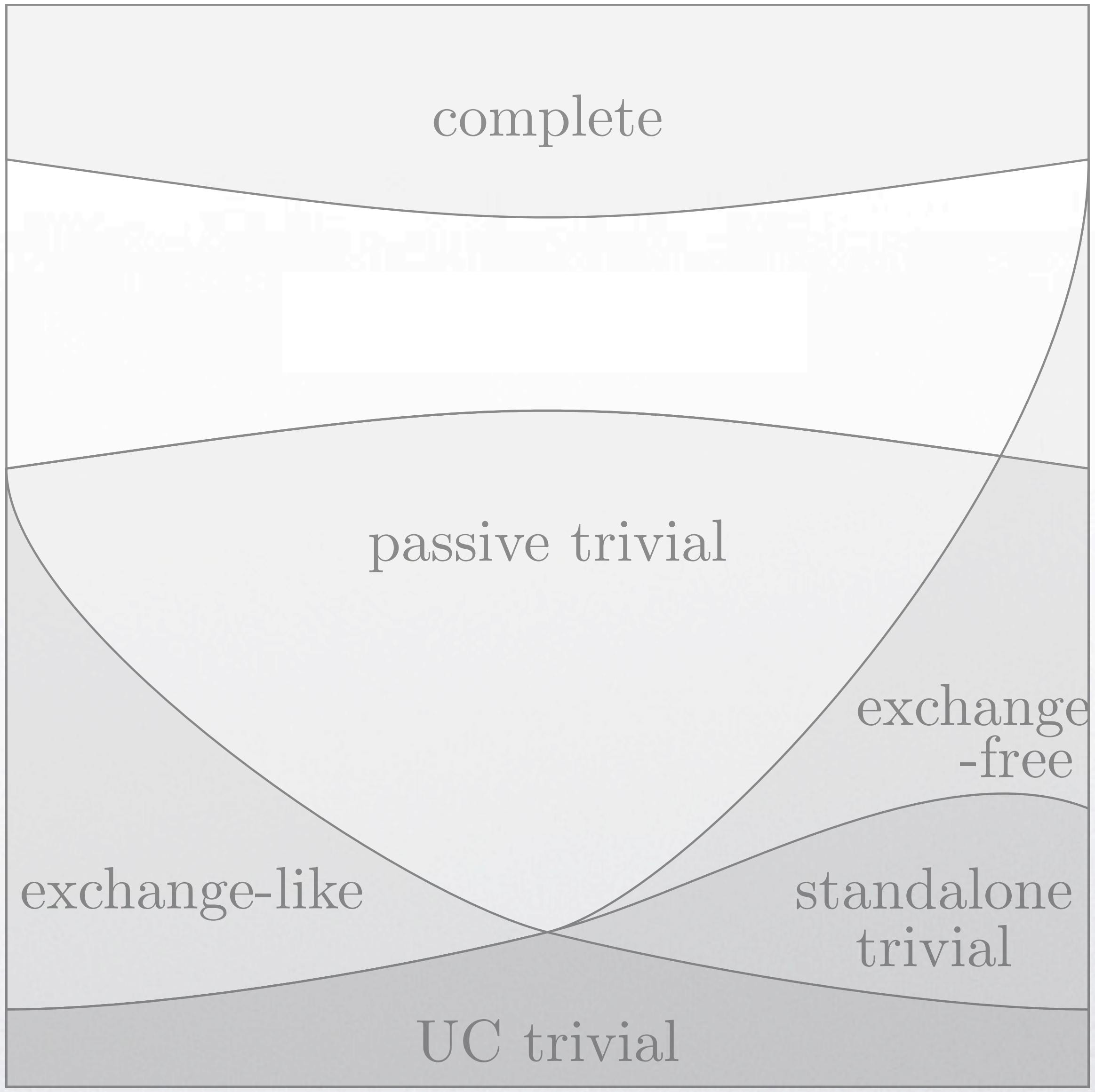
- So stopping the protocol at a random point gives the simulation an advantage over the honest strategy. Provides a “weak OT” that can then be amplified [DKS99]

shOT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passive trivial
exchange
-free
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trivial
UC trivial

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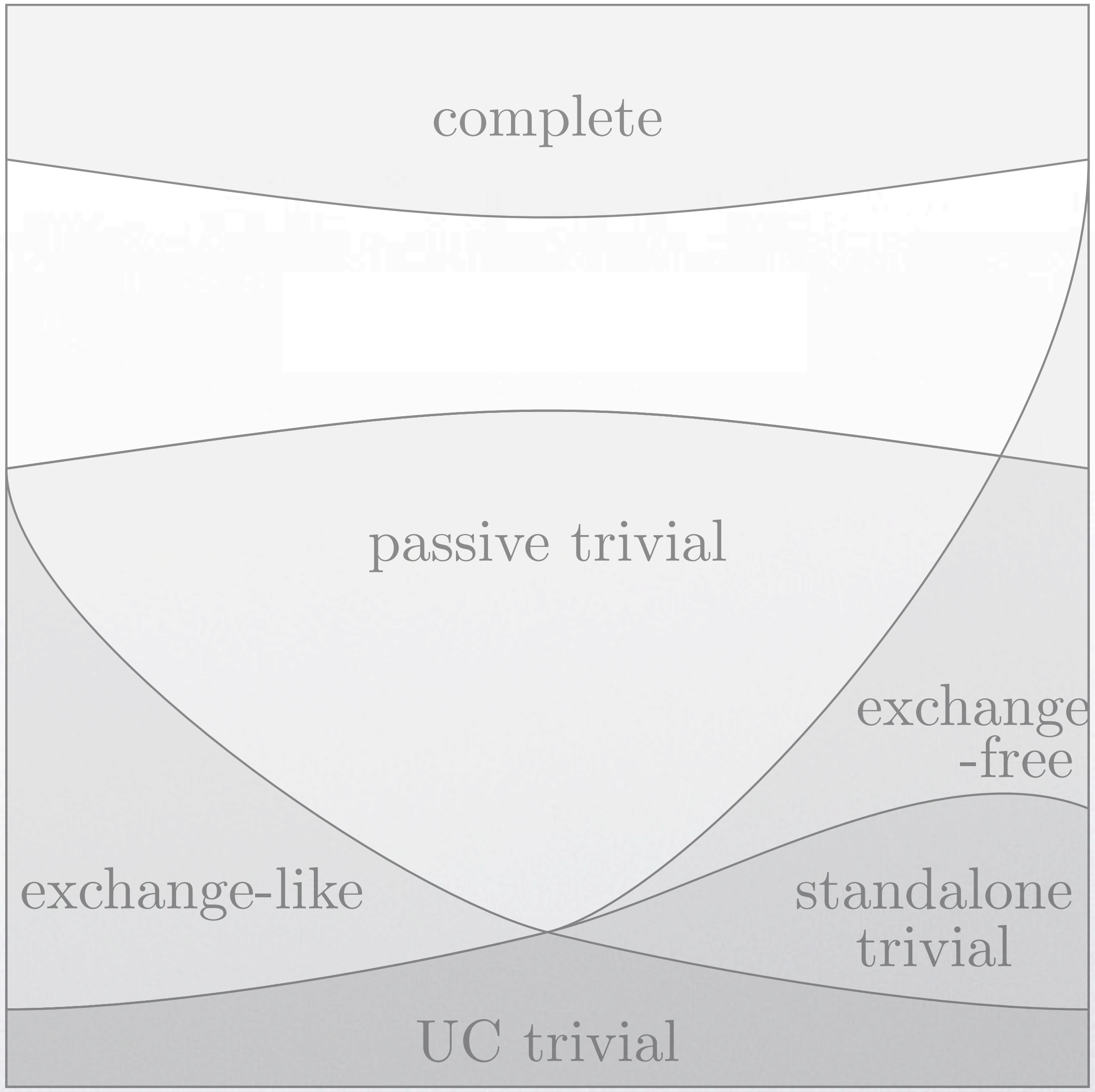

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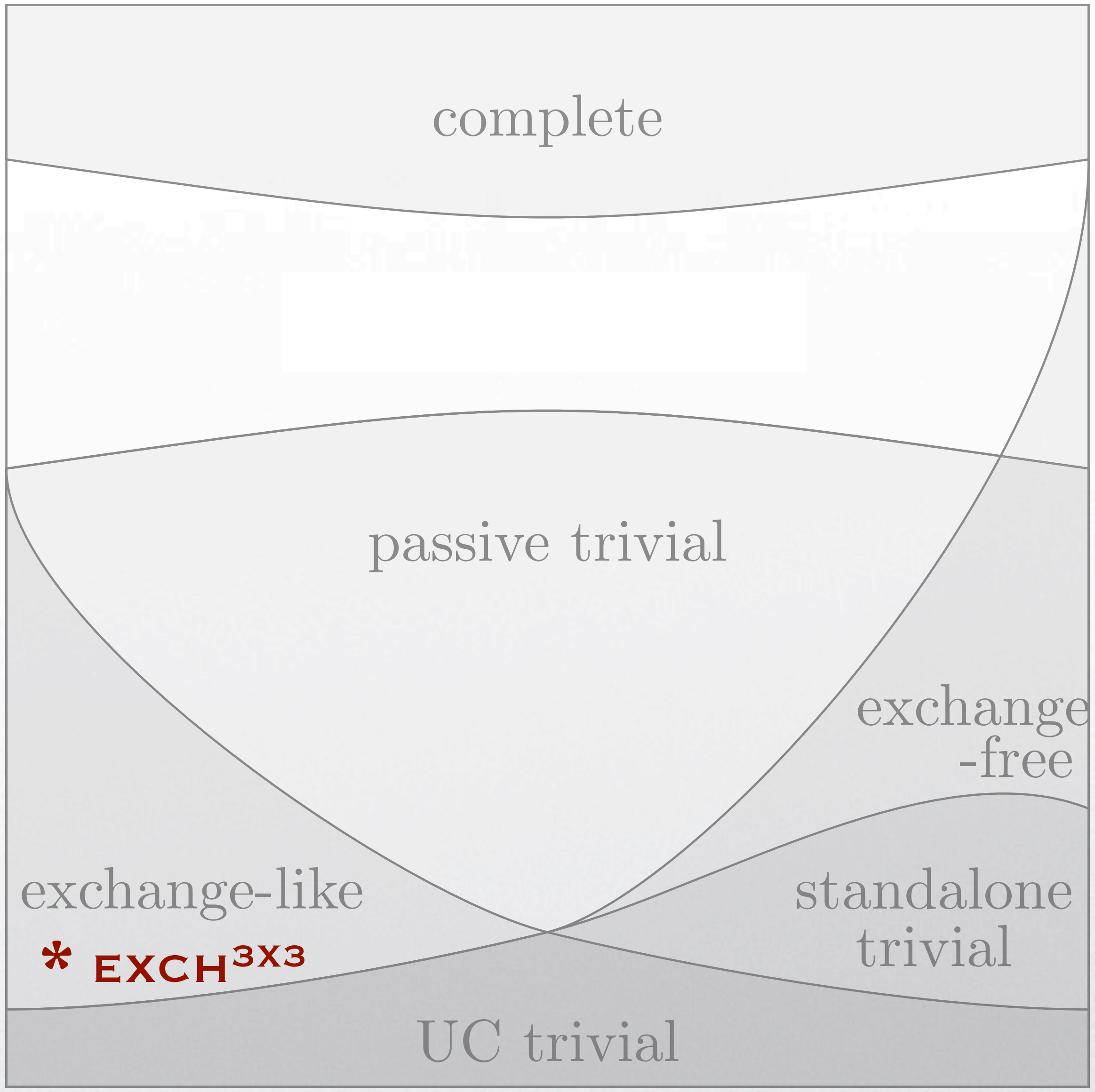

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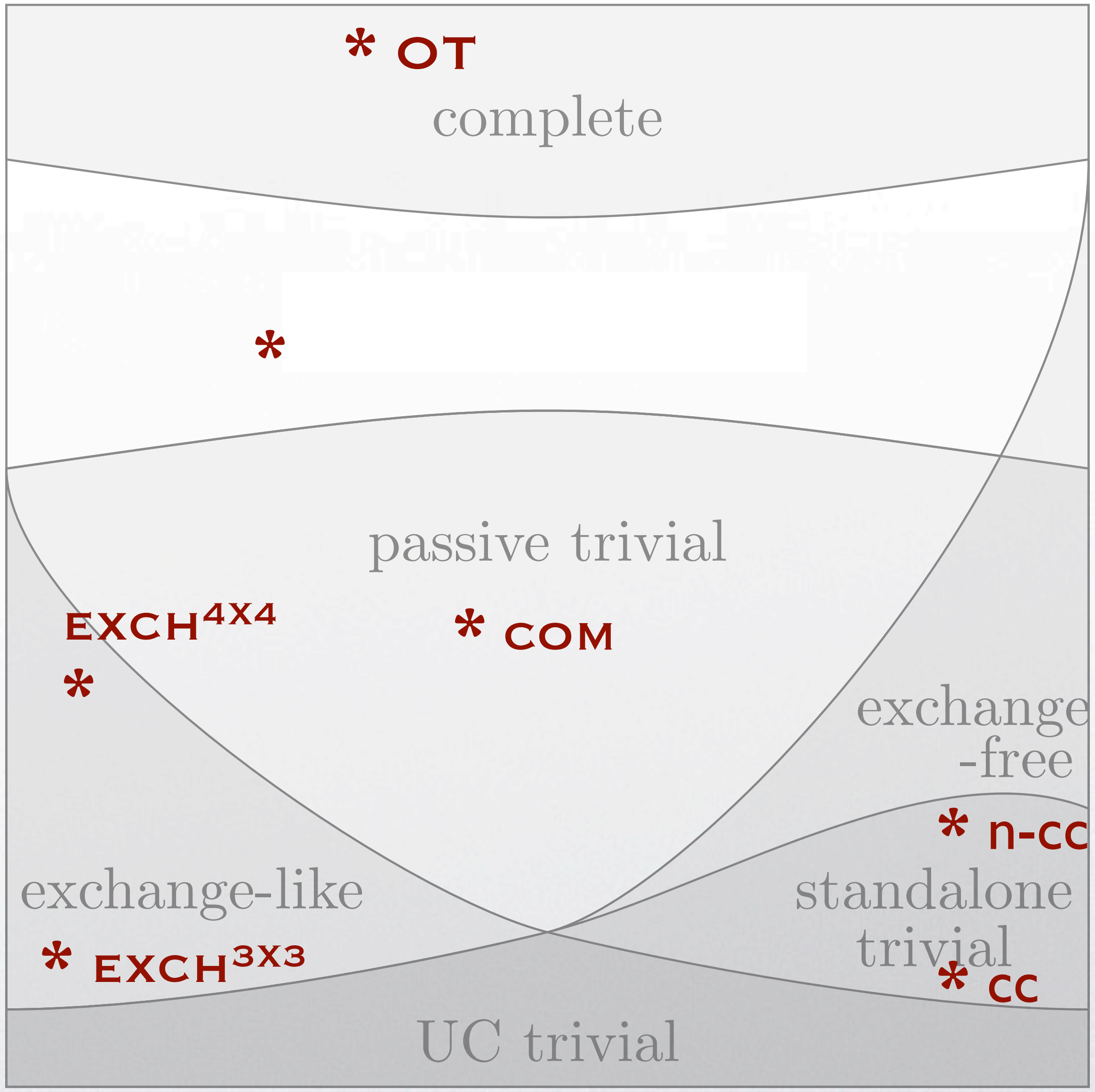

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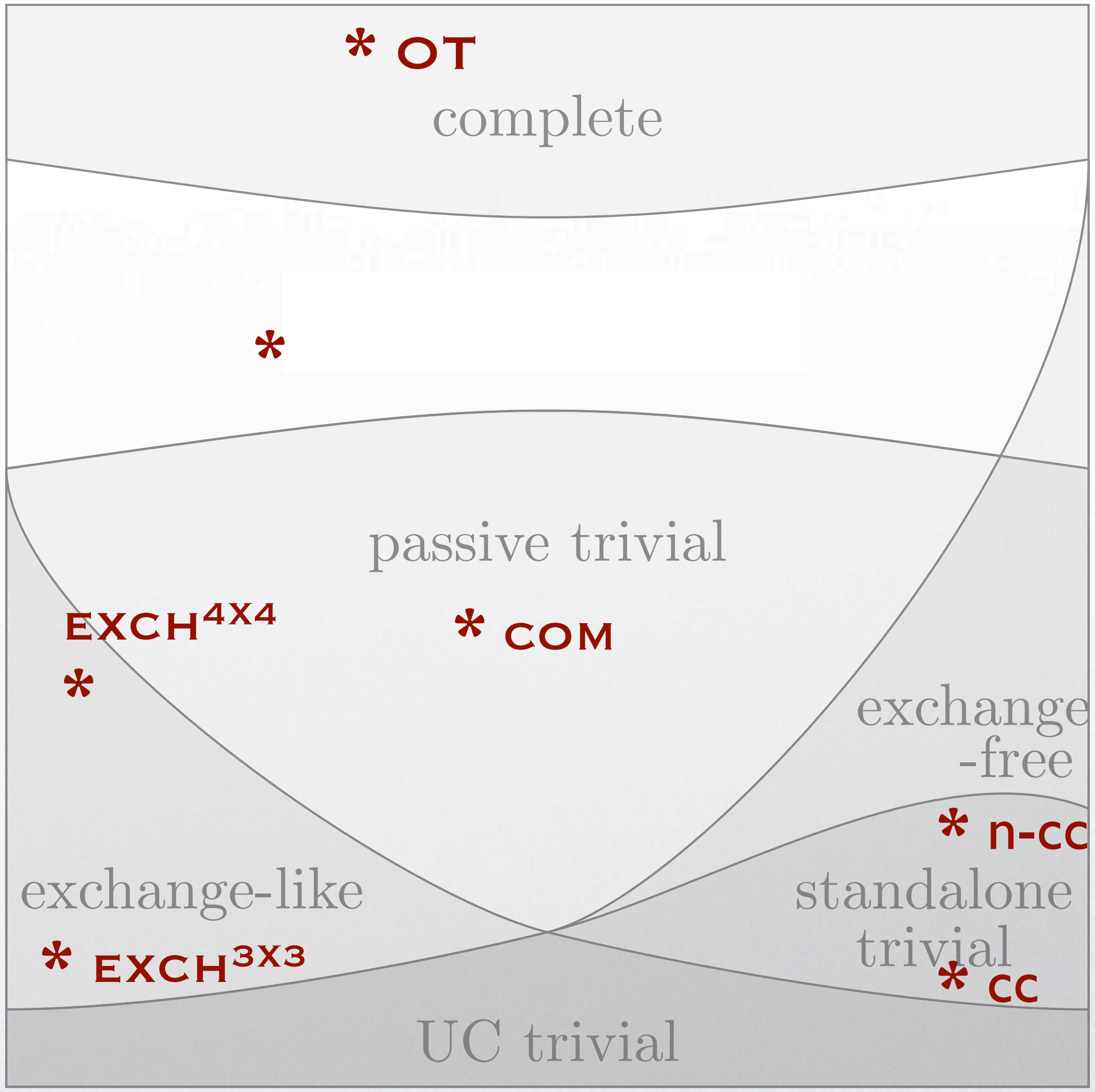

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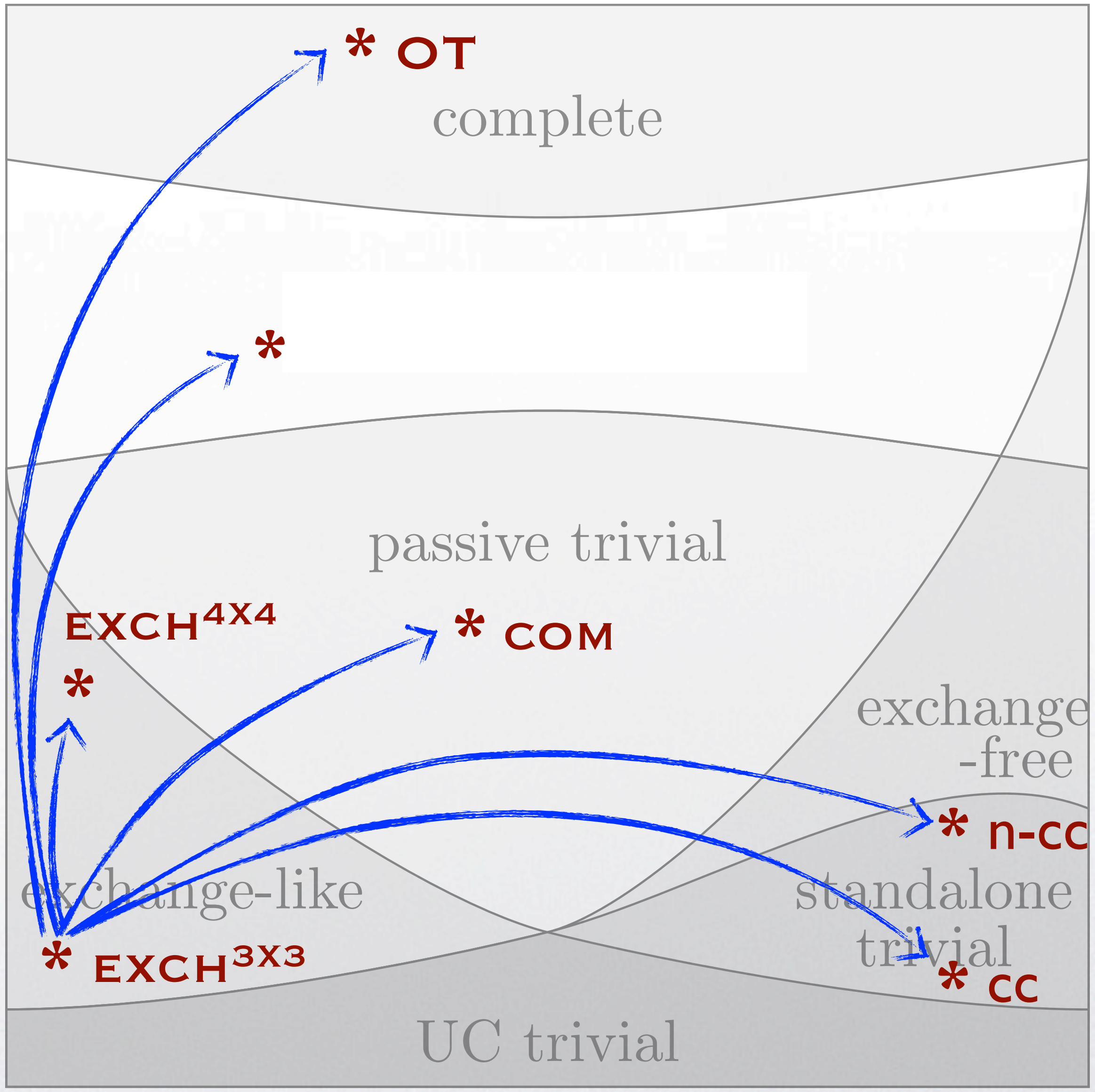

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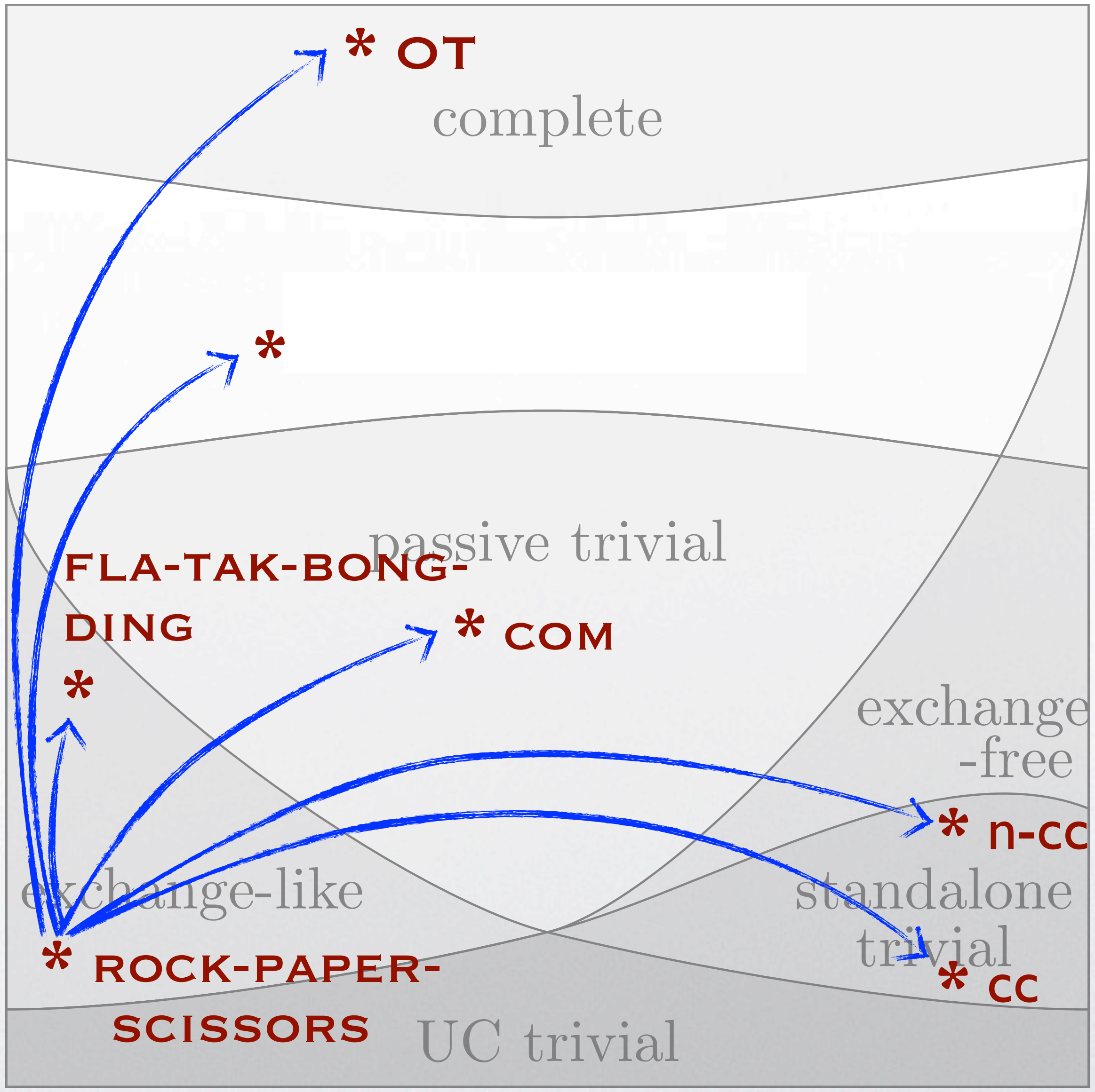

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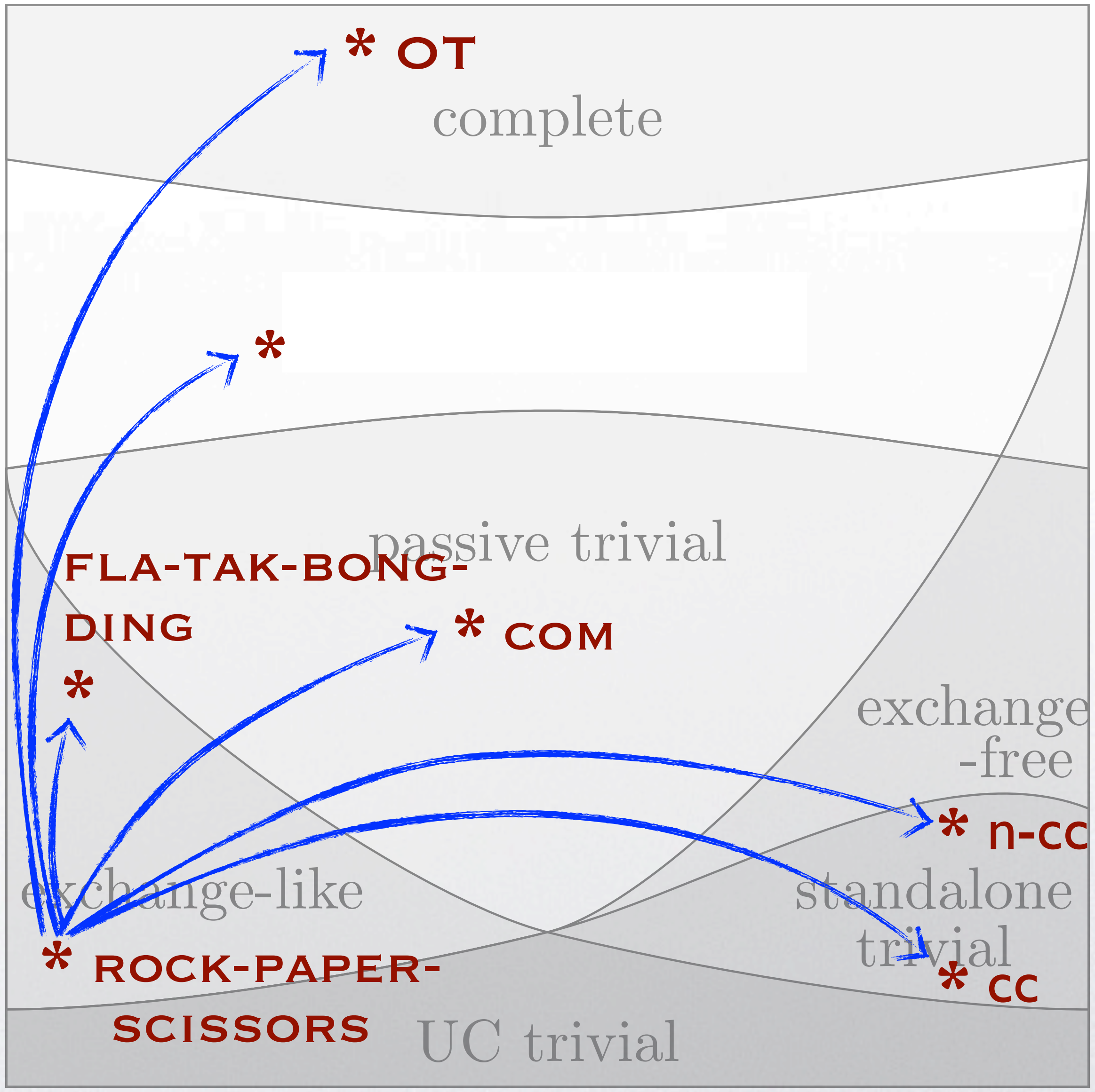

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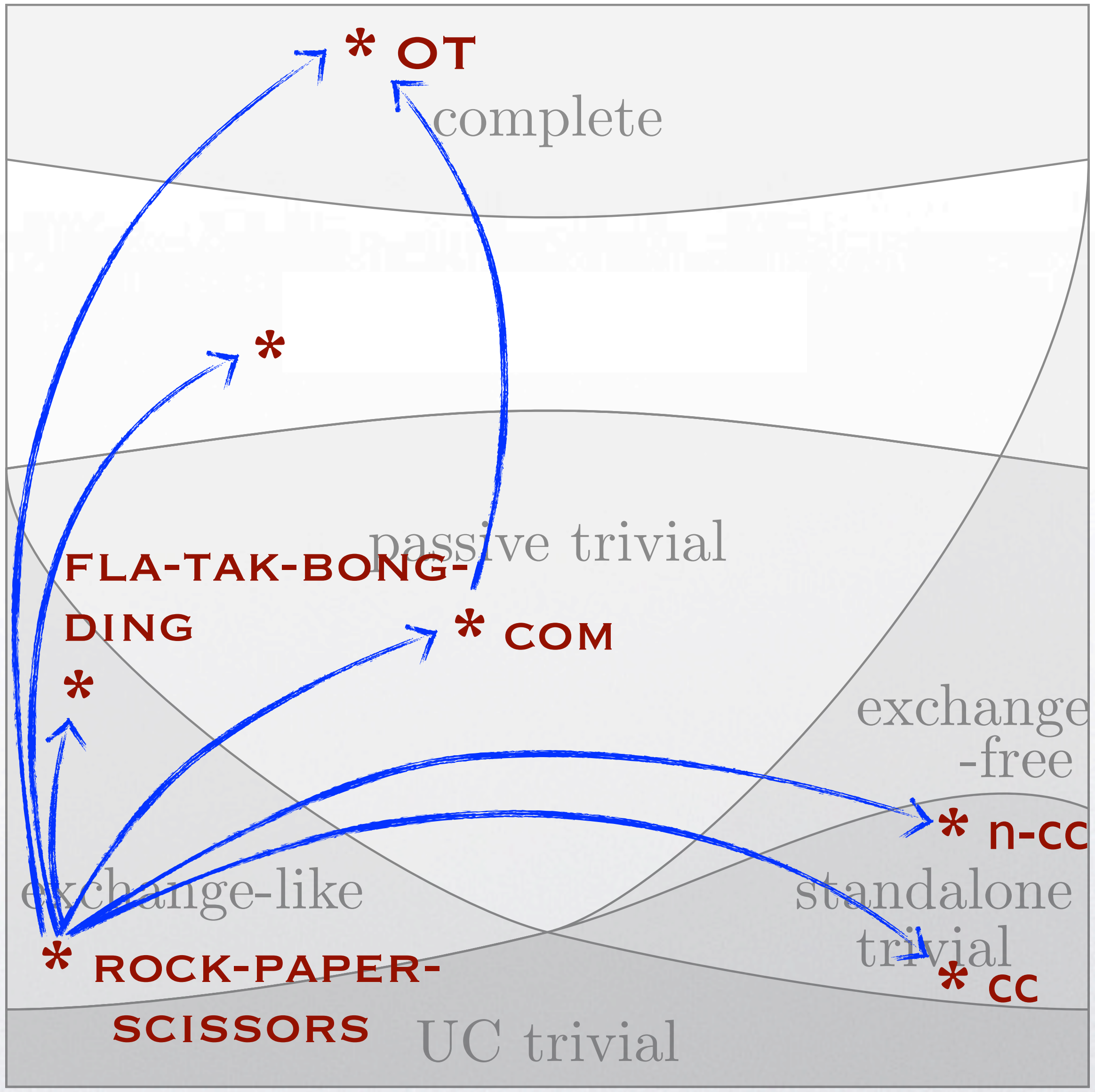

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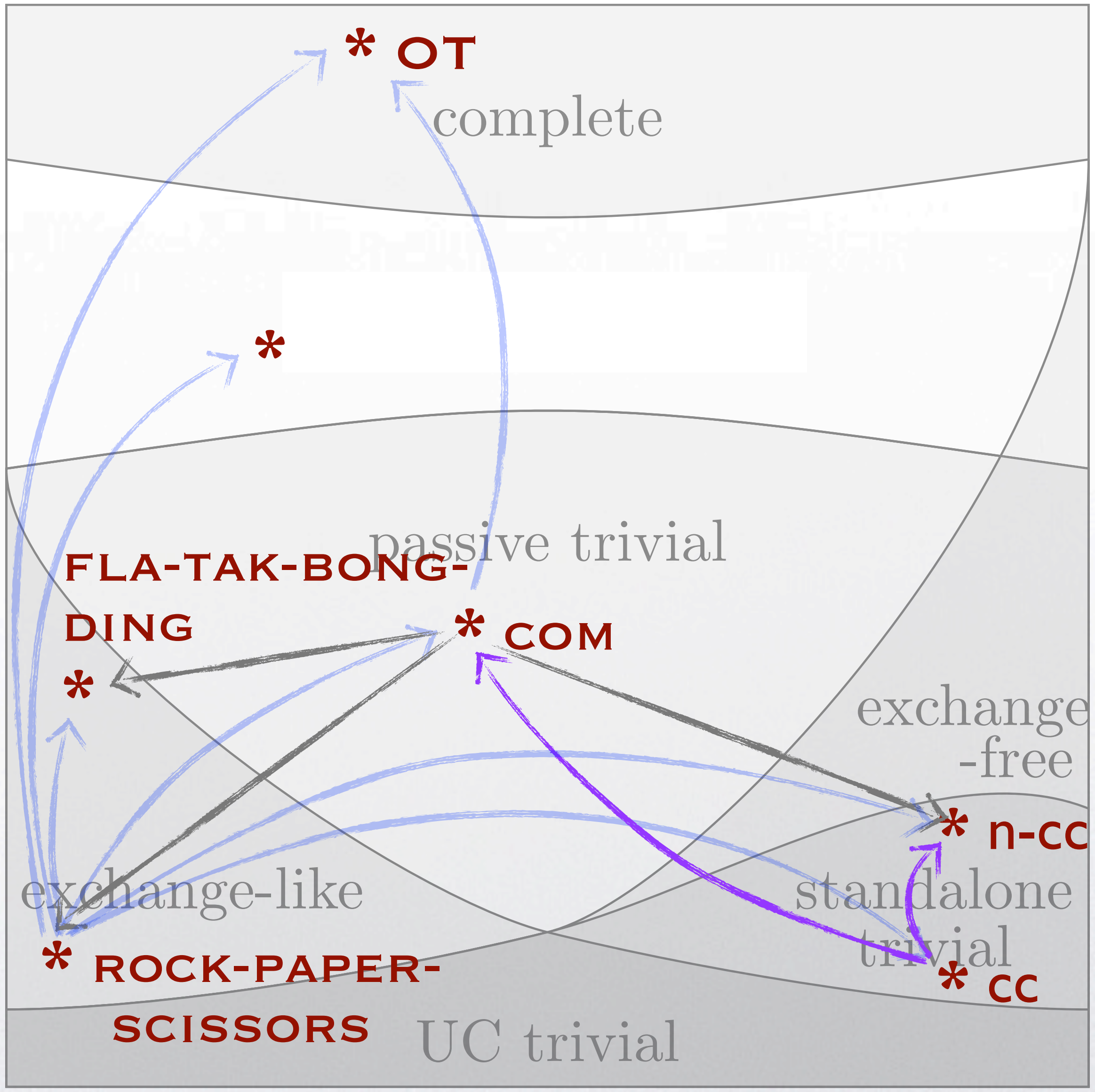
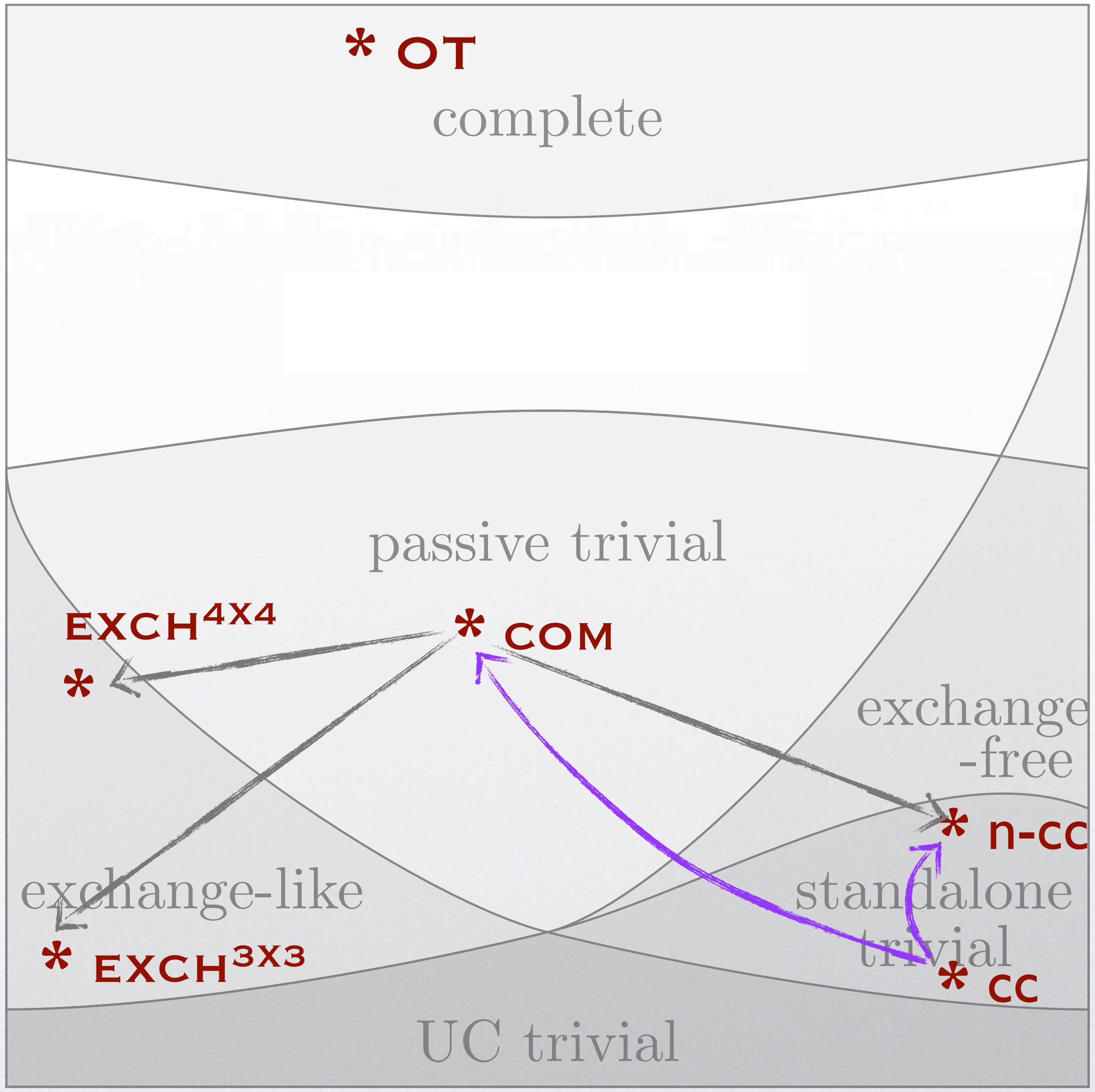
- All other reductions among "classified" F, G are implied by OWF (by results in [MPR09,MPR10b])



OWF


* OT
complete
passive trivial
EXCH^4X4
*
* COM
exchange
-free
* n-CC
exchange-like
* EXCH^3X3
standalone
trivial
* CC
UC trivial

OWF

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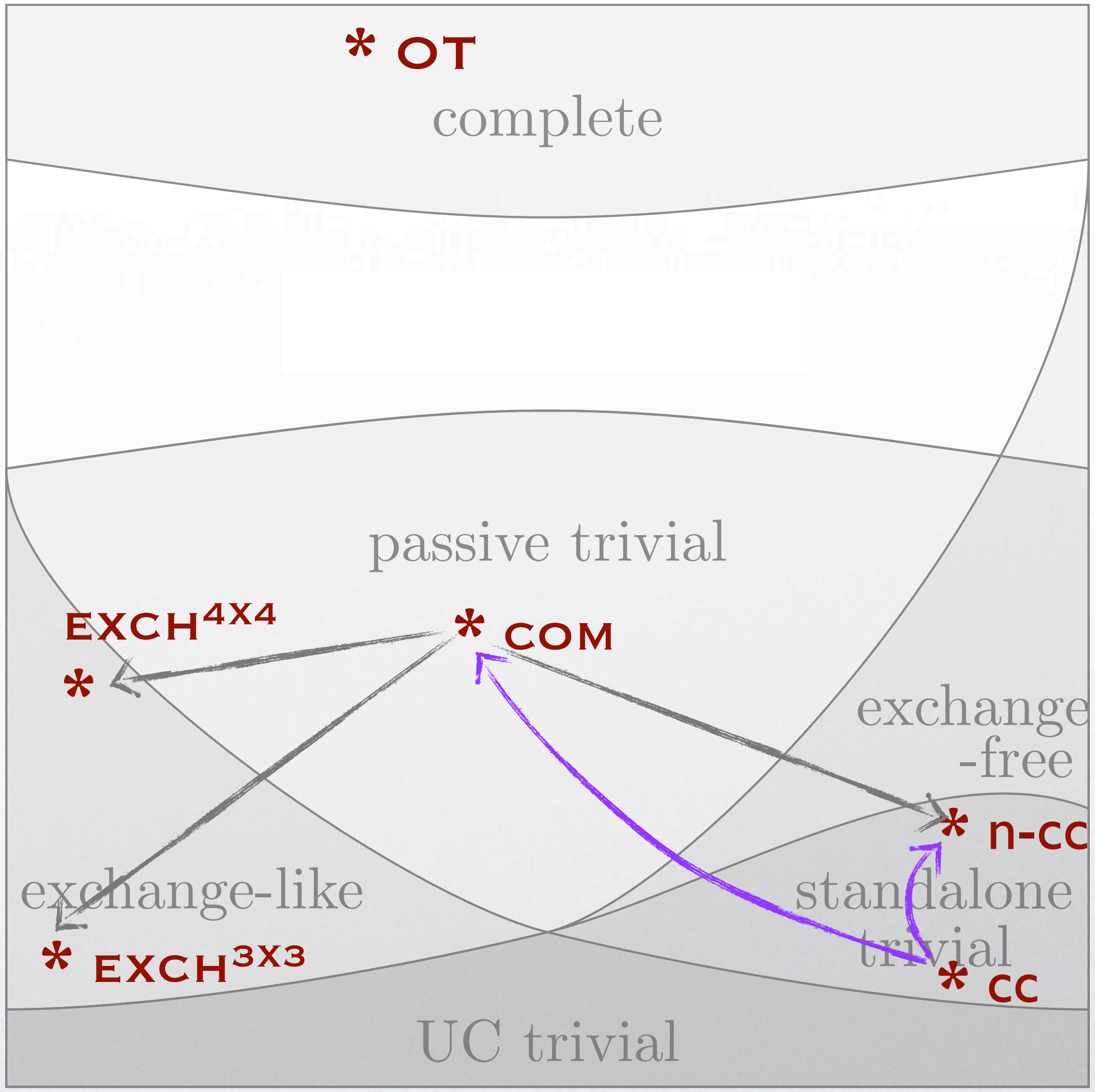

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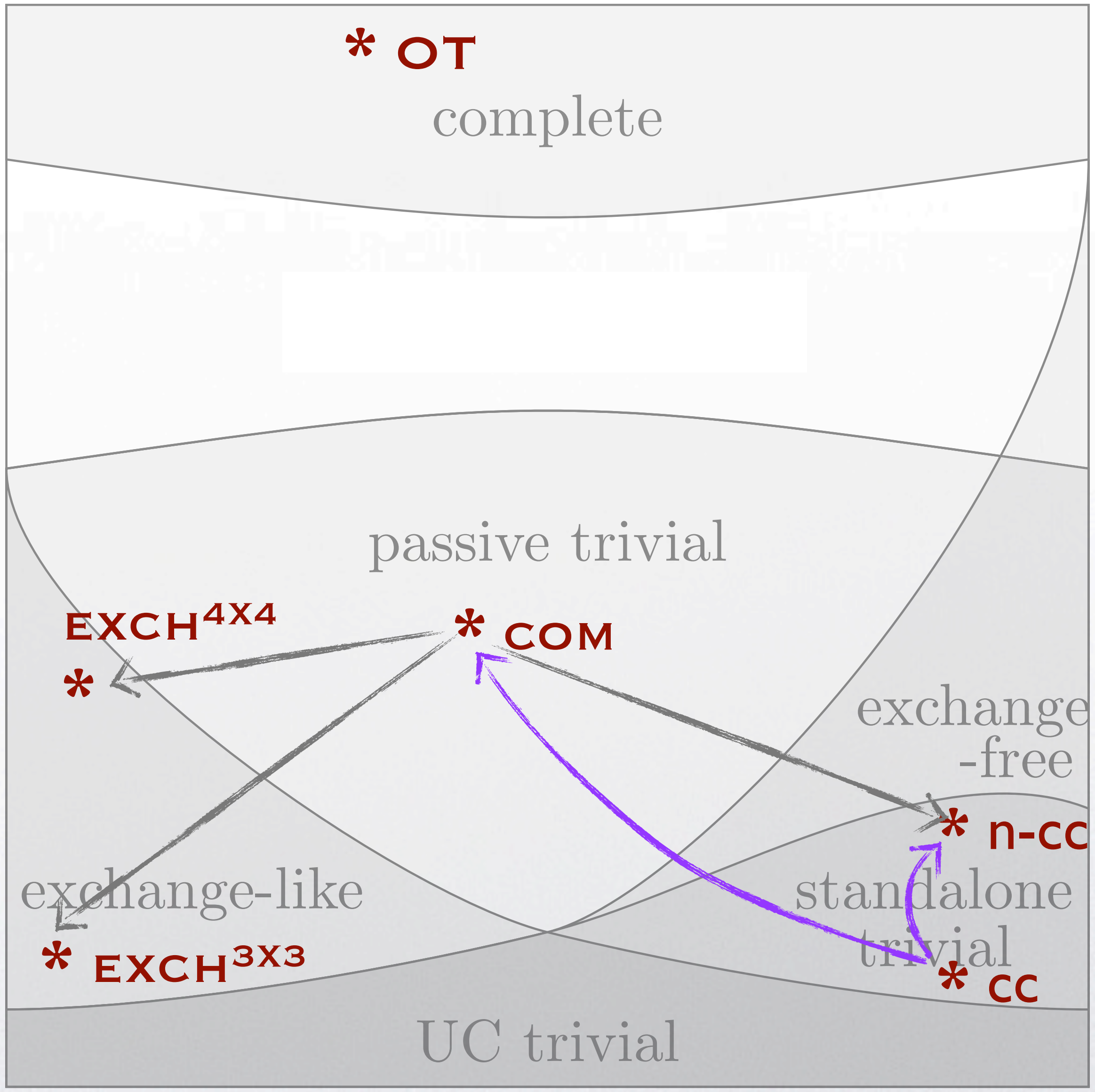

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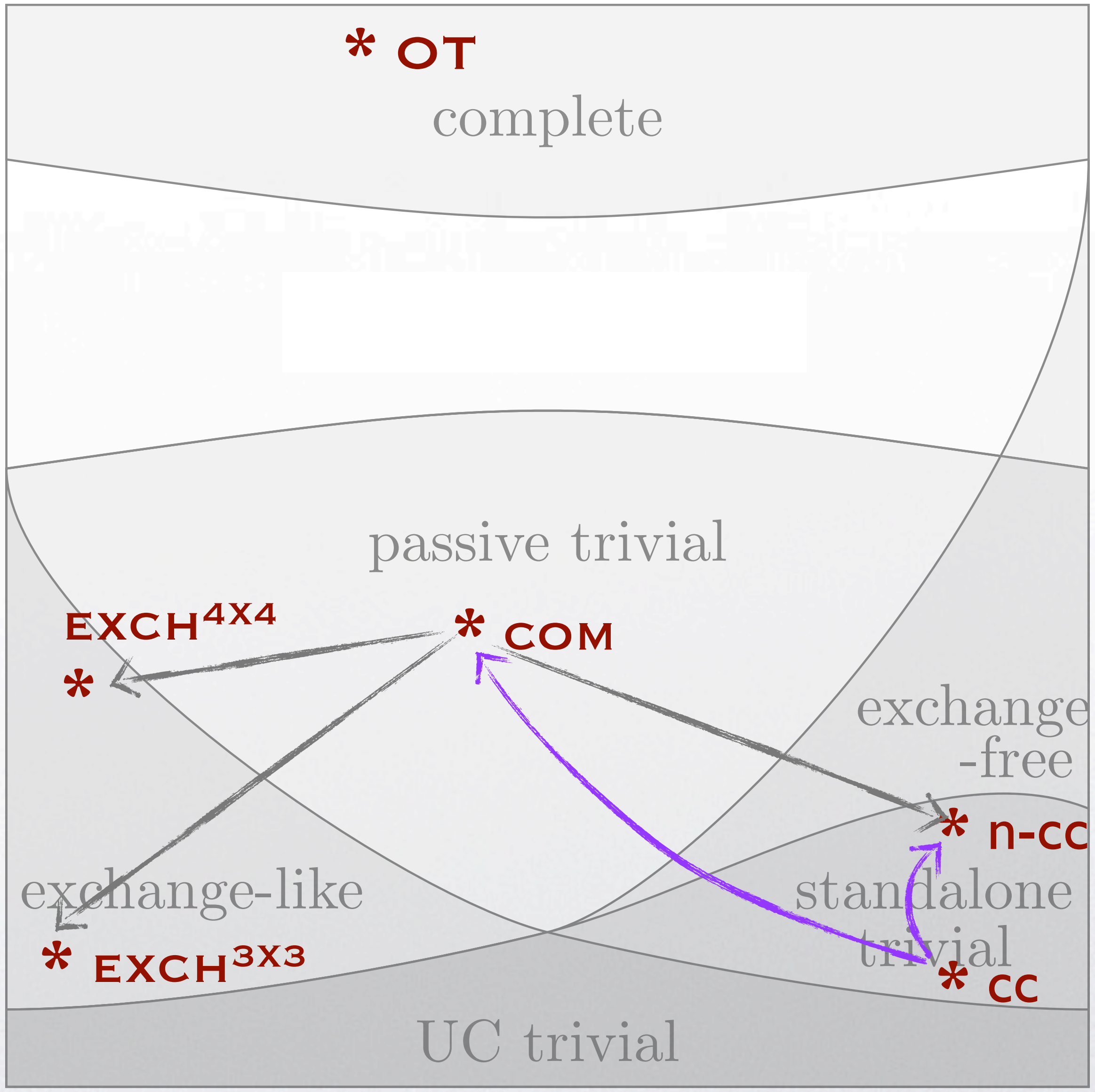
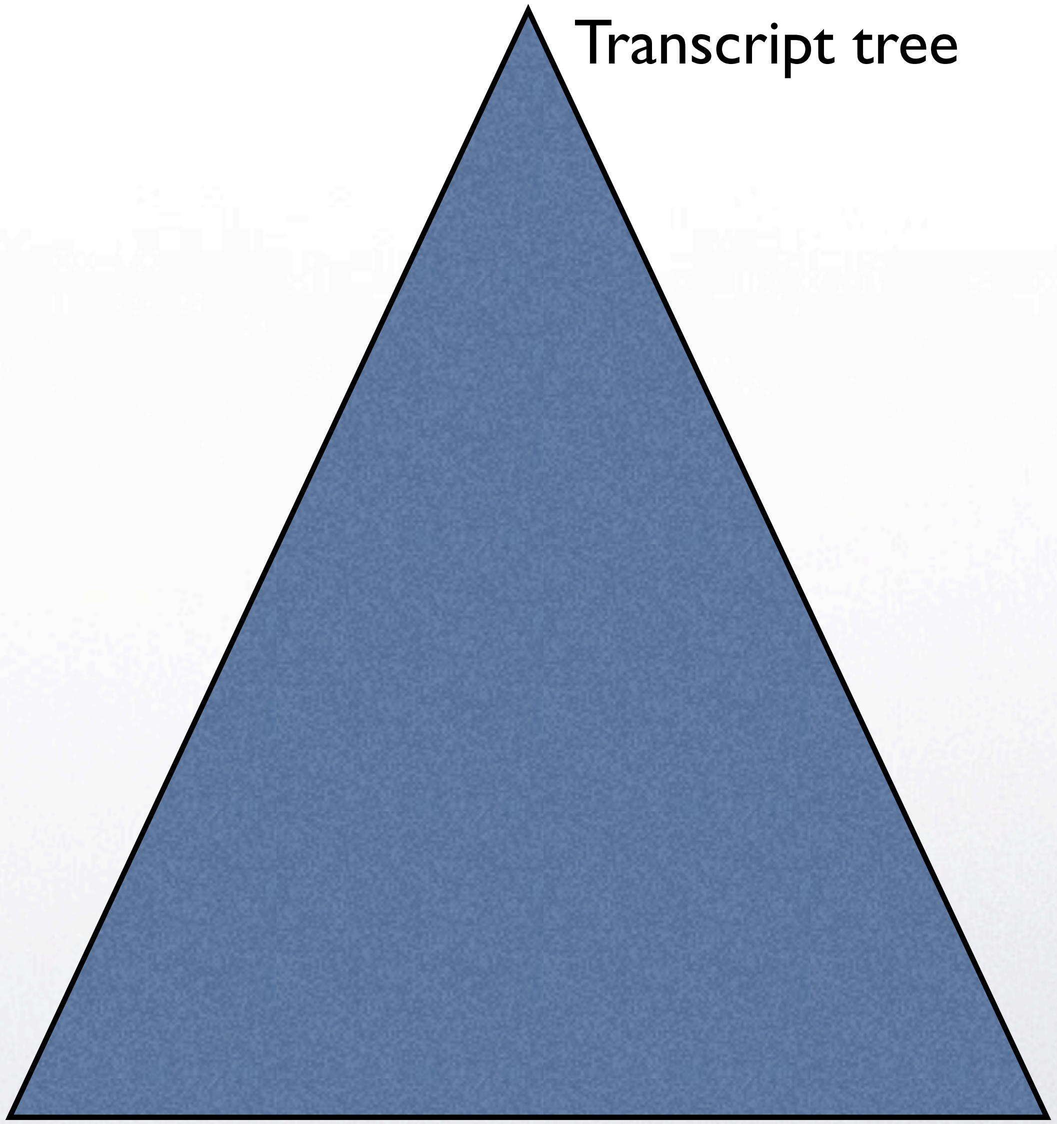
    - Frontier analysis: appears in [CI'93]. Reinvented (for other uses) in [MPR09], and used extensively in [MMOPR,MPS]

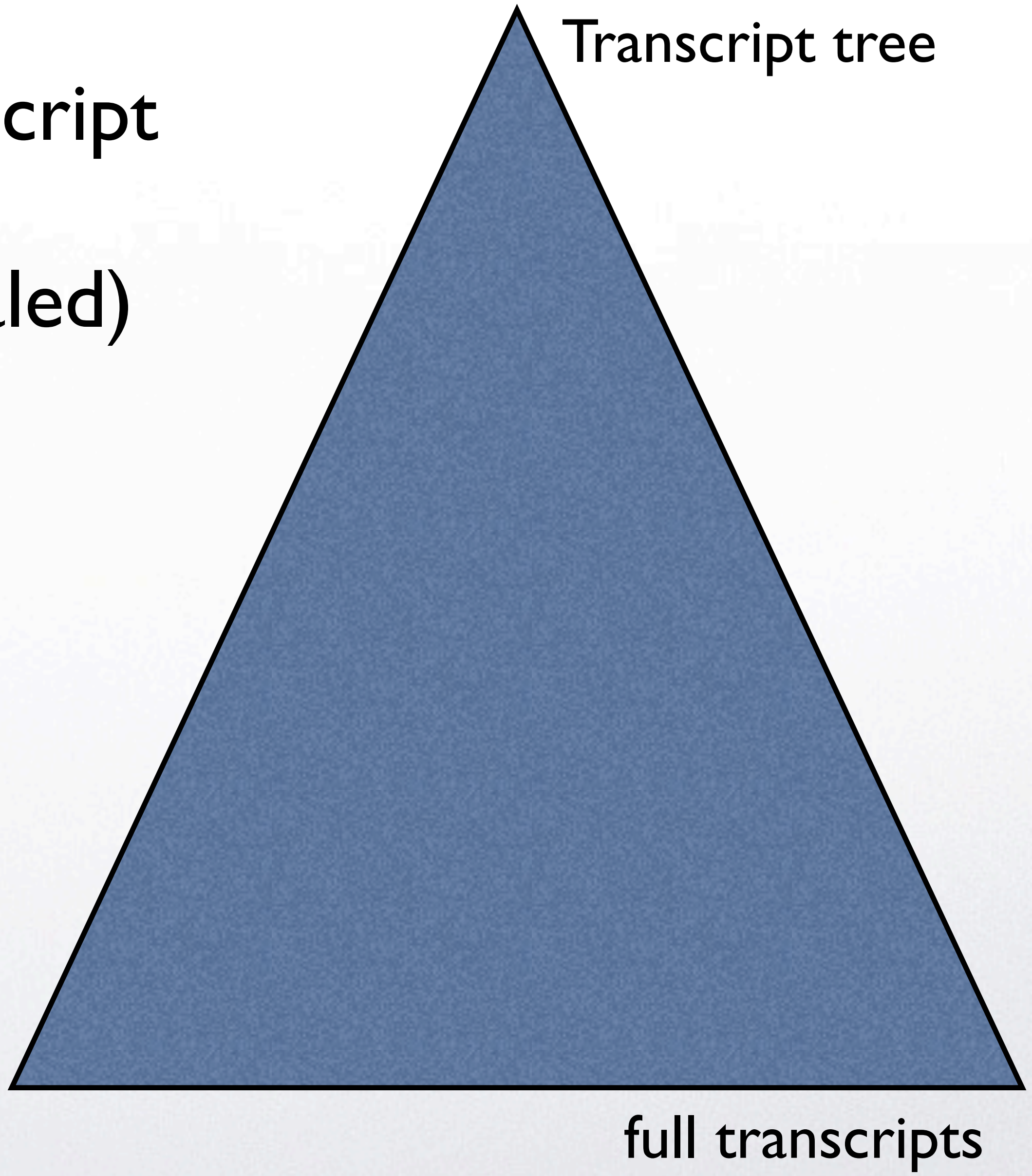


Frontier Analysis & OWF



Frontier Analysis & OWF

- Considers frontiers in a protocol's "transcript tree" where certain properties hold (e.g. some information about an input is revealed)



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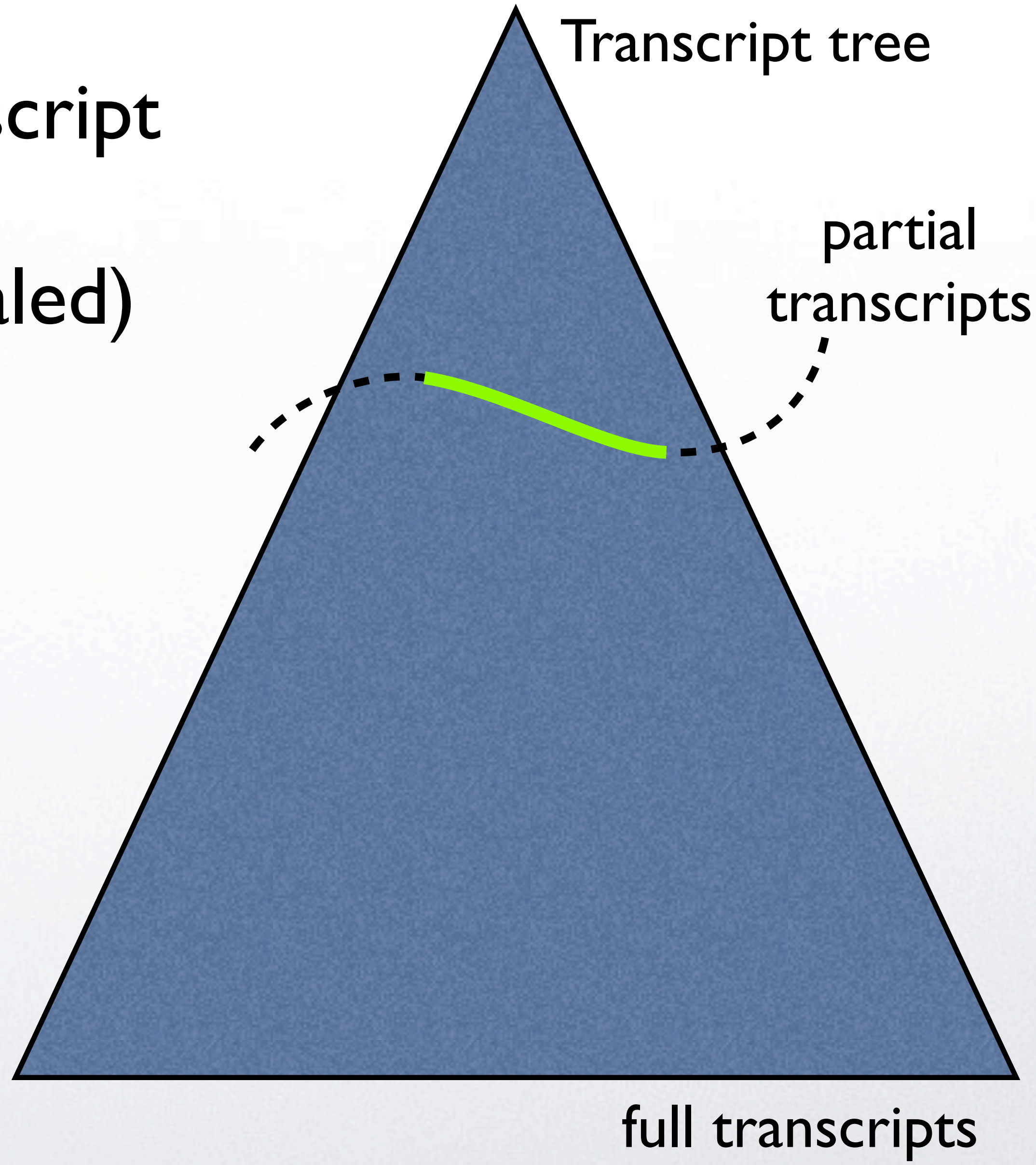

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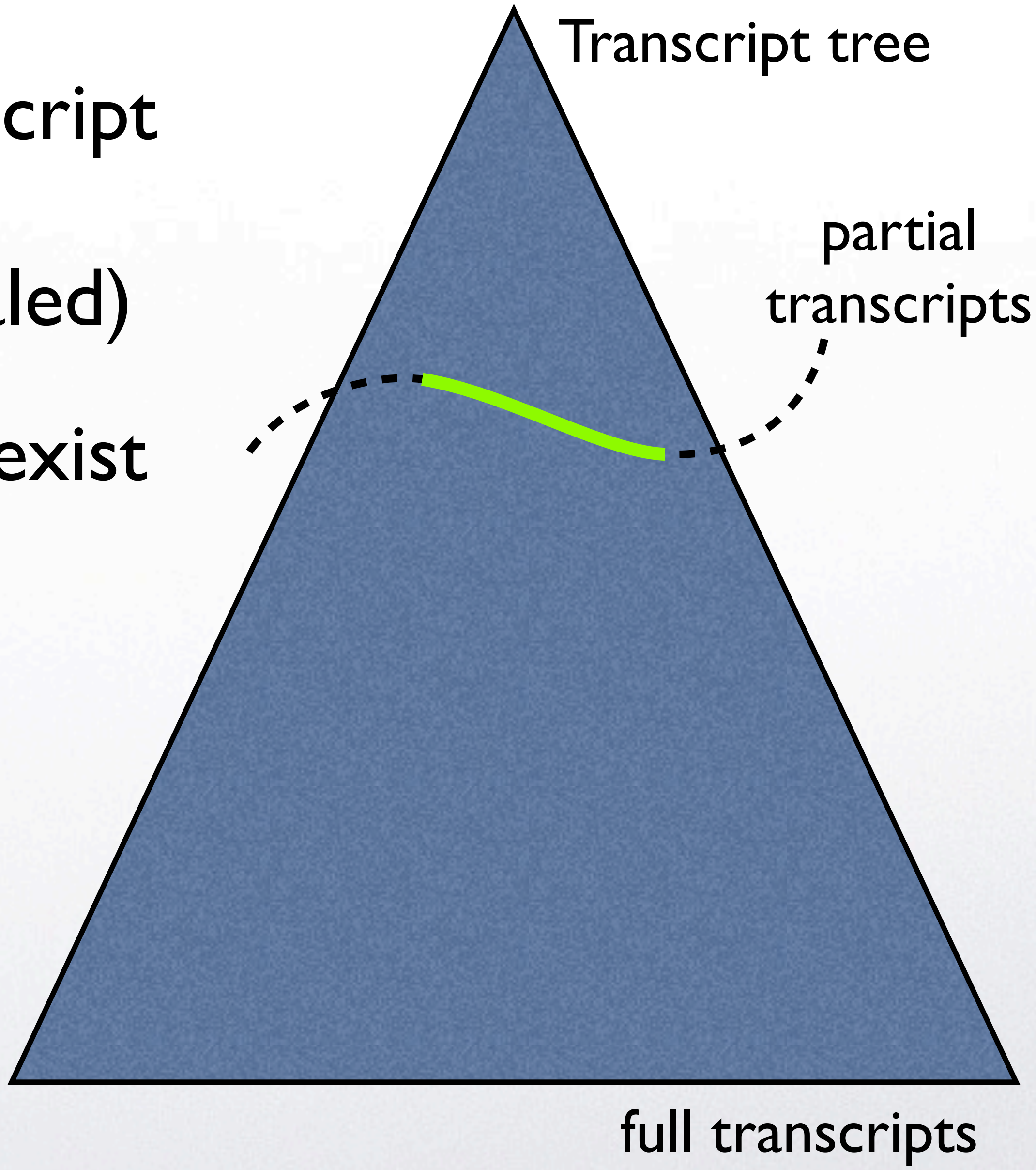

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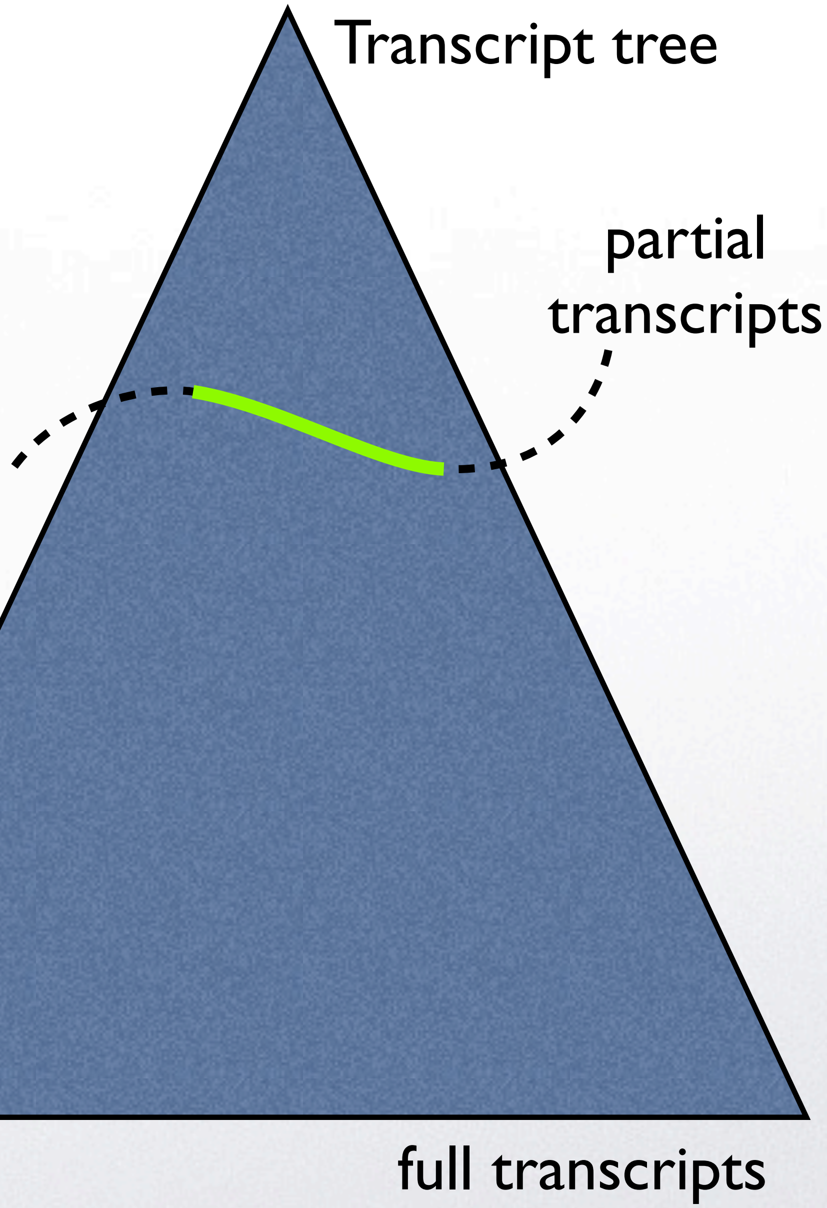

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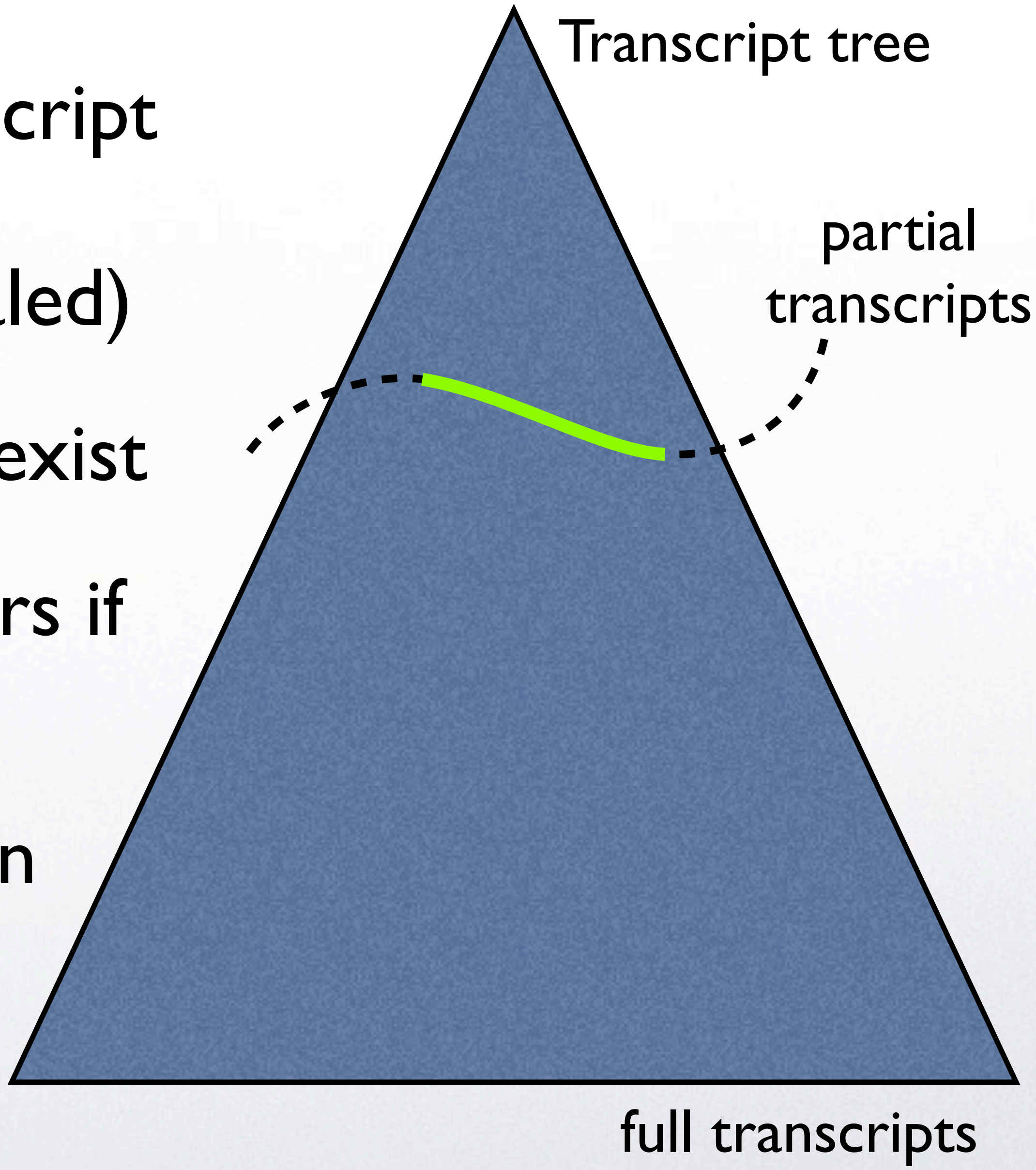
- Turns out, often, if OWFs don't exist, then can efficiently detect the frontiers (using characterization of OWF in [IL89])



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    - Even (statistical) cryptographic complexity little understood
  - Randomized functionalities, fair functionalities, infinite functionalities? (Again, cryptographic complexity little understood)

Crypto Means & Goals

One-Way Functions

One-Way Permutations

Trapdoor One-Way Permutations

OT protocol

Collision-Resistant Hash Functions

Zero-Knowledge Proofs

Encryption

Signatures

Homomorphic Encryption

Secret Communication Channels

Authenticated Communication Channels

OT Channel

Digital Cash

Mental Poker

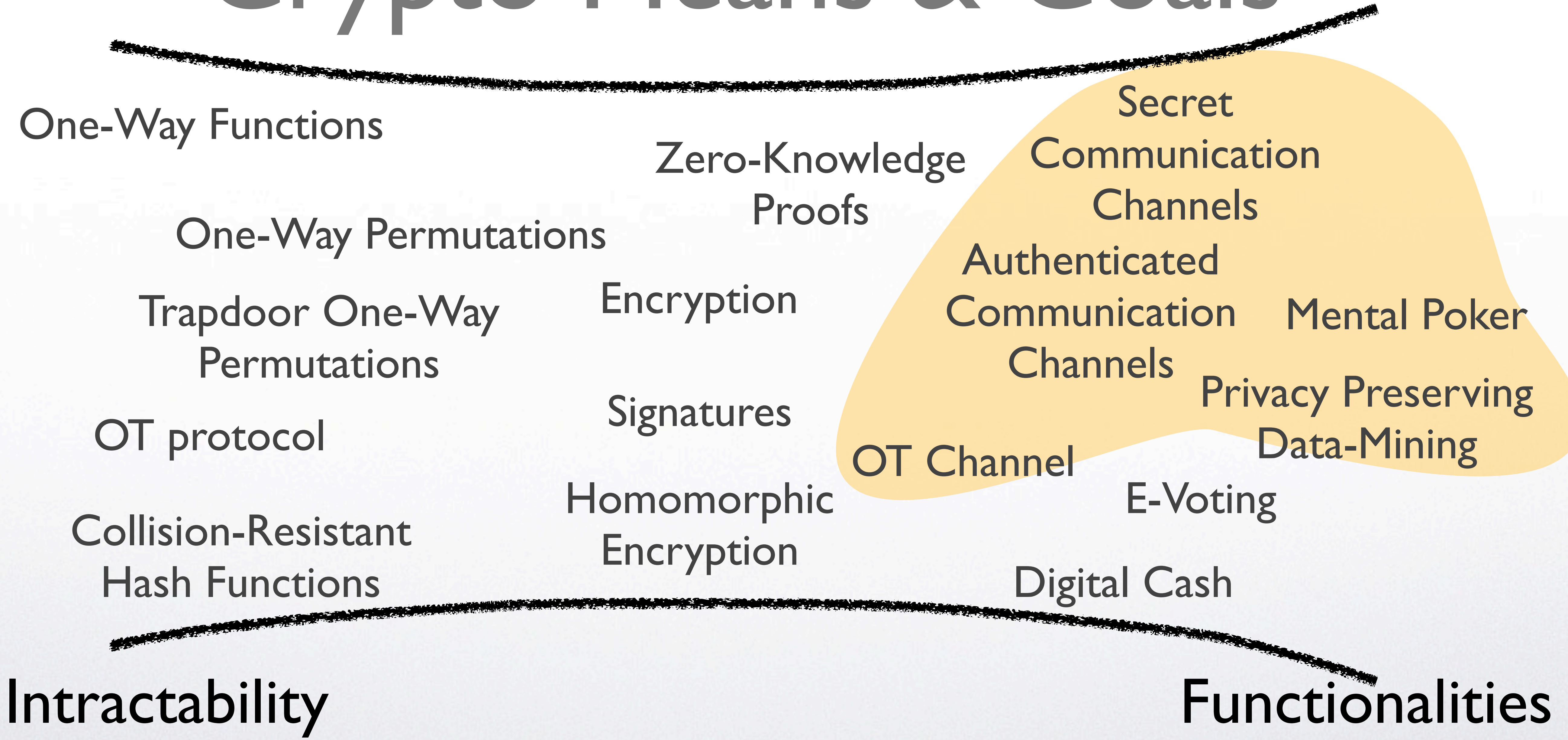
Privacy Preserving Data-Mining

E-Voting

Intractability

Functionalities

Crypto Means & Goals



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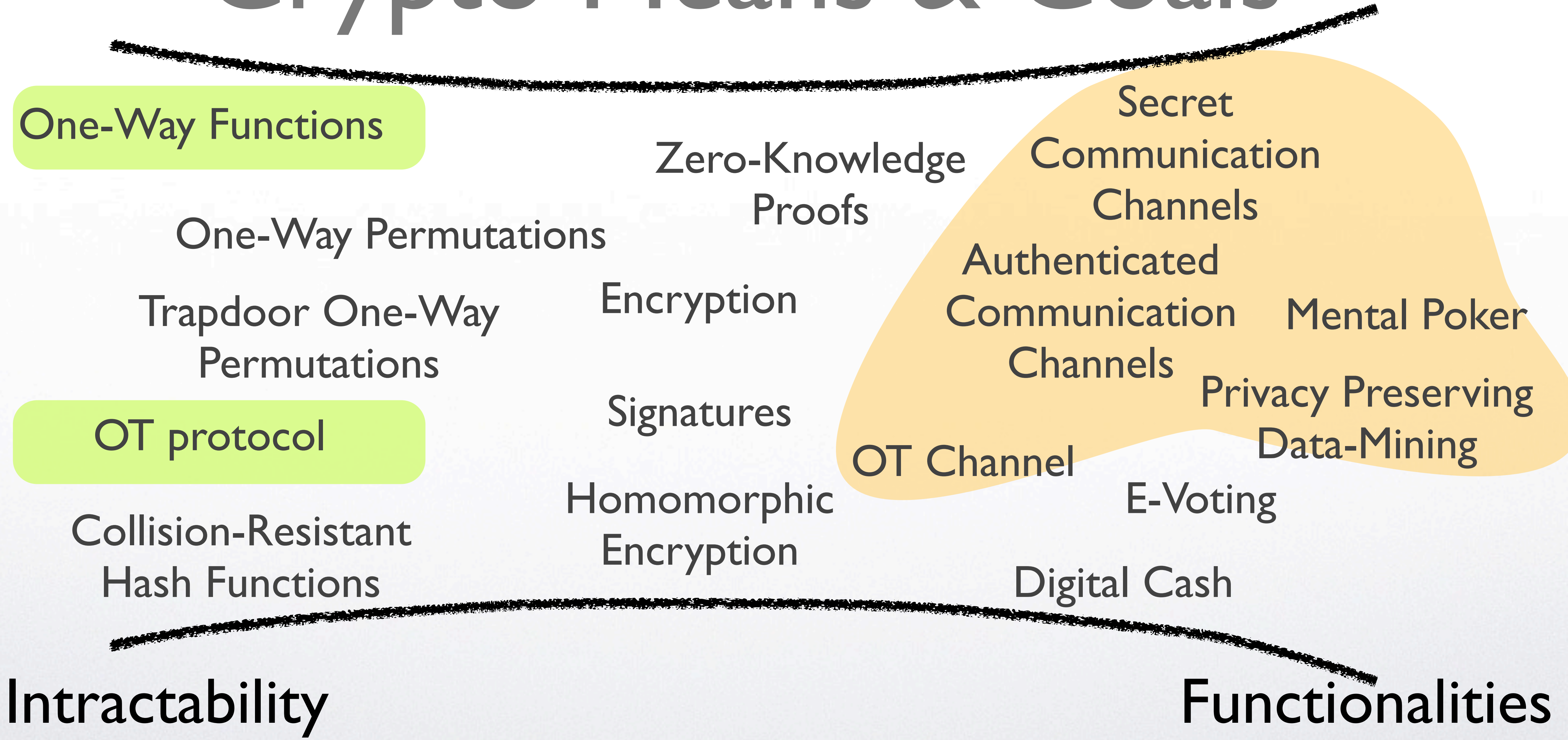

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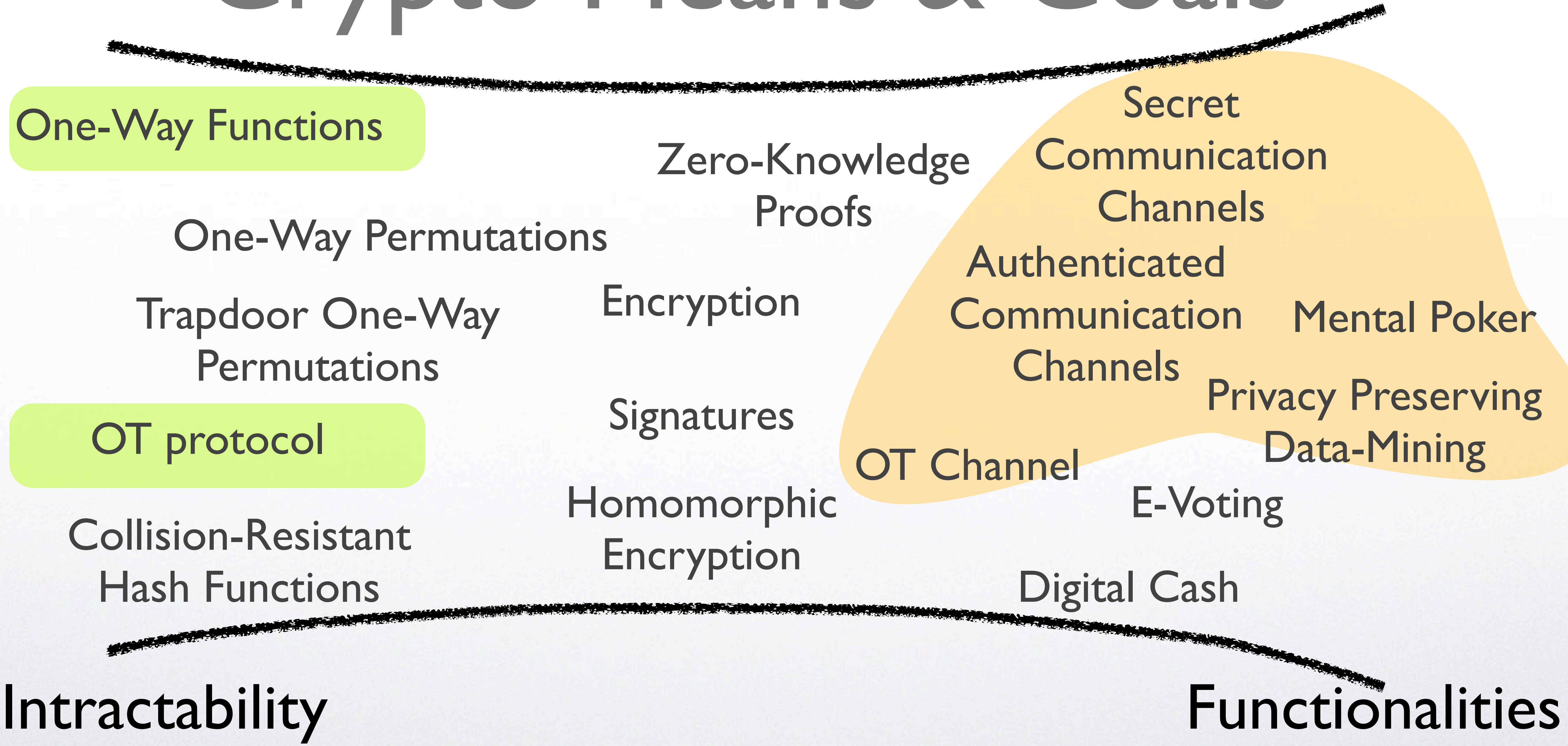

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