Circumventing the Price of Anarchy Leading Dynamics to Good Behavior

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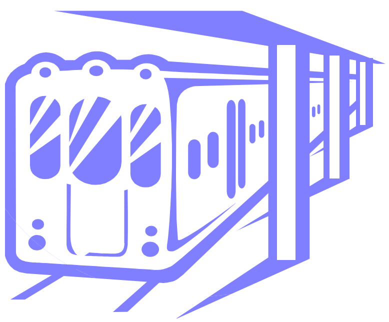
Joint with Avrim Blum and Yishay Mansour

Good equilibria, Bad equilibria

Many games have both bad and good equilibria.

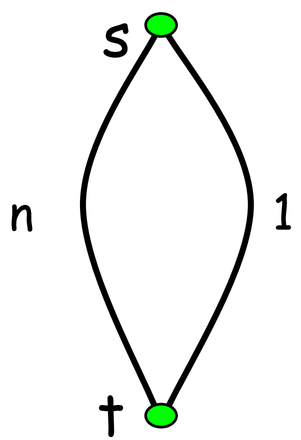
- In some places, everyone drives their own car. In some, everybody uses and pays for good public transit.



Good equilibria, Bad equilibria

Fair cost-sharing

- n players in directed graph G, each edge e costs c_e.
- Player i wants to get from s_i to t_i.
- all players share cost of edges they use with others.
- $\mathrm{cost}(s) = \sum_{i=1}^{n} \mathrm{cost}_i(s)$



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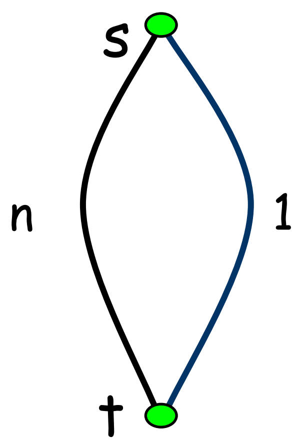

Good equilibrium: all use edge of cost 1.

(paying 1/n each)

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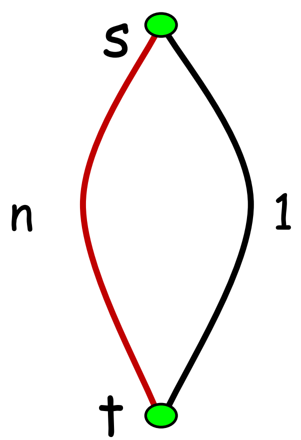


Good equilibrium: all use edge of cost 1.

(paying 1/n each)

Bad equilibrium: all use edge of cost n.

(paying 1 each)

Inefficiency of equilibria, PoA and PoS

Price of Anarchy (PoA): ratio of worst Nash equilibrium to OPT.

[Koutsoupias-Papadimitriou'99]

Price of Stability (PoS): ratio of best Nash equilibrium to OPT.

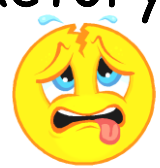
[Anshelevich et. al, 2004]

E.g., for fair cost-sharing, PoS is log(n), whereas PoA is n.

Significant effort spent on understanding these in CS.

Dynamics in Games

- Traditionally: convergence to some equilibria
  - Best/better response
  - Regret Minimization
  - Imitation Dynamics
- Not so satisfactory in games with a huge gap between PoA and PoS



What can we say about getting to good states?

Getting to Good Equilibria

I) Players entering one at a time

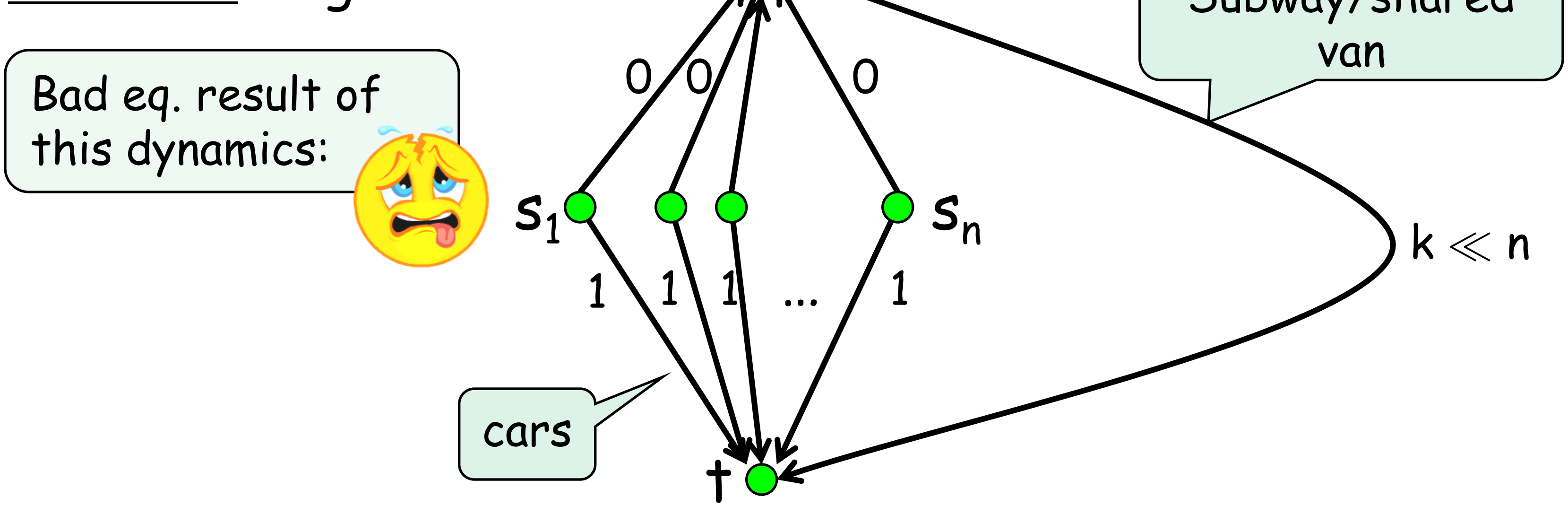
- Undirected single sink fair cost sharing, one at a time entering from an empty config. [Charikar et al, 2008]
- Positive result; get within polylog(n) factor of OPT
- But fails in directed graphs.

Getting to Good Equilibria

I) Players entering one at a time

- <u>Undirected</u> single sink fair cost sharing, one at a time entering from an empty config. [Charikar et al, 2008]
- Positive result; *get* within polylog(n) factor of OPT

<u>Directed</u> single sink



Getting to Good Equilibria

II) Noisy best response (simulated annealing on potential function)

[Blume95, Marden/Shamma08, Young05]

$\mathrm{Pr}_i(a) \propto e^{-\mathrm{cost}_i(s_a)/\tau}$ [Prob. of action a decreases exponentially with gap between the cost of a and cost of BR]

Show examples of directed cost sharing where no noisy- best-response alg can do better than POA within poly #of steps.

How can we get around this

Analyze if a helpful entity/source encourage (guide) behavior to move from a bad state to a good state.

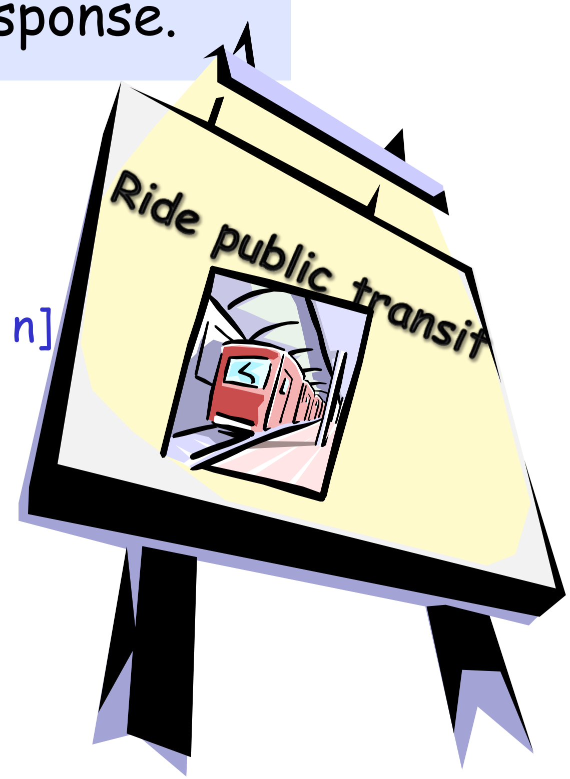
Getting to Good Equilibria

III) Public Service Advertisement [BBM, SODA 2009]

- A helpful authority advertises a good joint action.
- A random constant fraction of the players follow the proposal temporarily; others do best response.

Strong positive result for fair cost sharing

If α fraction of players follow the advice, then get within $O(1/\alpha)$ of PoS. [PoS = log(n), PoA = n]



Note: The model requires:

- receptive/gullible players
- non-receptive/stubborn players.

What if each player is a bit of both?

Our Proposed Model: High level

A more adaptive model

Each player has a few abstract actions.

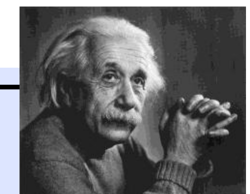
Uses a learning, experts based alg. to decide which one to use

Expert 1

Play Best Response

Expert 2

Play the Proposed Behavior



[no rigid separation between receptive vs non-receptive players]

Our Model

Begin in some arbitrary configuration.

Someone analyzing game comes up with a good idea (joint action of low cost) and proposes it.

Players go in a random order:

With probability p_i do proposed action.

With probability $1-p_i$ do best-response to current state.

Model A: p_i's stay fixed, at some poly time T^*, everyone commits one way or the other. [Learn then Decide]

Model B: Players use arbitrary learning rule to slowly vary their p_i's. (only limit is learning rate). [Smoothly Adaptive]

What will happen to the overall state of the system?

Our Results

Fair Cost Sharing

Proposed action = OPT.

Learn then Decide

A poly number exploration of steps T^* is sufficient s.t. the expected cost at any time $T \geq T^*$ is $O(\log(n)\log(nm)OPT)$.

Smoothly Adaptive

$n_i = \Omega(m)$, $p_i \geq \beta$ for poly steps, then $\exists$ $T^* = \mathrm{poly}(n)$ s.t. whp cost at any time $T \geq T^*$ is $O(\log(nm)OPT)$.

Consensus

For any graph, any initial configuration, if $p_i > \frac{1}{2}$ then whp play will reach optimal in $O(n \log^2 n)$ steps.

Fair Cost Sharing

Key Lemma #1:

So long as all $p_i \geq \epsilon$ for constant $\epsilon > 0$, whp the cost will reach $O(OPT \cdot \log(mn))$ within poly(n) steps.

Proof sketch preliminaries:

- Fair cost-sharing -- exact potential game: $\exists$ potential fnc Φ s.t. if any player makes a move decreasing their own cost by Δ, then Φ drops by Δ too.

$$S = (P_1, P_2, \ldots, P_n) \quad \text{cost}(S) = \sum_{e \in \cup_i P_i} c_e$$

$$\phi(S) = \sum_e \sum_{x=1}^{n_e} f_e(x) \quad \text{where} \quad f_e(x) = \frac{c_e}{x}$$

- For any state S, $\text{cost}(S) \leq \Phi(S) \leq \text{cost}(S)\log(n)$.

Fair Cost Sharing

Key Lemma #1:

So long as all $p_i \geq \epsilon$ for constant $\epsilon > 0$, whp the cost will reach $O(OPT \cdot \log(mn))$ within poly(n) steps.

Proof sketch:

- After initial startup phase, whp all edges e with $n_e \gg \log(nm)$ players on them in OPT, will have $\geq (\epsilon/2)n_e$ players on them now.
- Implies OPT is a "fairly good" response for everyone (cost $O(\log(nm)OPT_i)$, where OPT_i = i's cost in OPT).
- So, if cost is currently high, if player i picked at random, expected drop in Φ is large (whether i does proposed action or BR).
- Can't happen for too long (use martingale tail bound).

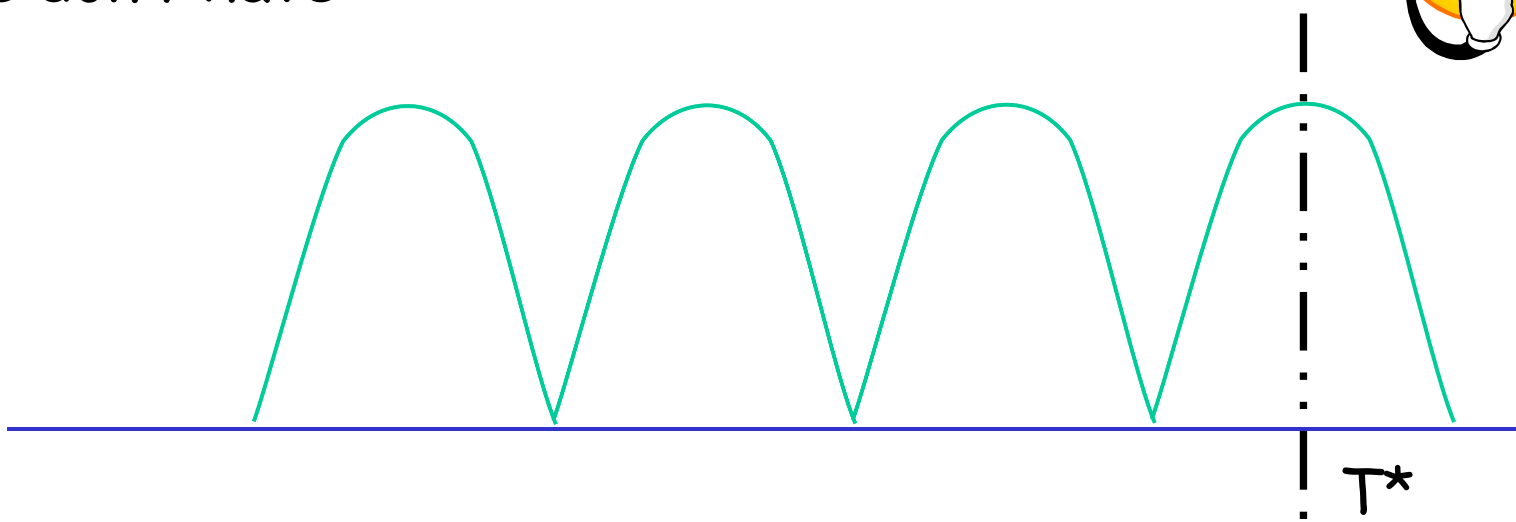
Fair Cost Sharing

Key Lemma #1:

So long as all $p_i \geq \epsilon$ for constant $\epsilon > 0$, whp the cost will reach $O(OPT \cdot \log(mn))$ within $poly(n)$ steps.

Great - cost gets low pretty soon! 

But not quite enough to get what we want…need to ensure don't have:



Fair Cost Sharing, Learn then Decide

Key Lemma #1:

So long as all $p_i \geq \epsilon$ for constant $\epsilon > 0$, whp the cost will reach $O(OPT \cdot \log(mn))$ within $poly(n)$ steps.

Key Lemma #2:

So long as all $p_i \geq \epsilon$ for constant $\epsilon > 0$, if cost at time T_1 is $O(OPT \cdot \log(mn))$, then $E[\Phi]$ at any time $T = T_1 + poly(n)$ is $O(OPT \cdot \log(mn) \cdot \log(n))$.

Final step for learn then decide model:

In final decision step, potential cannot increase by much.

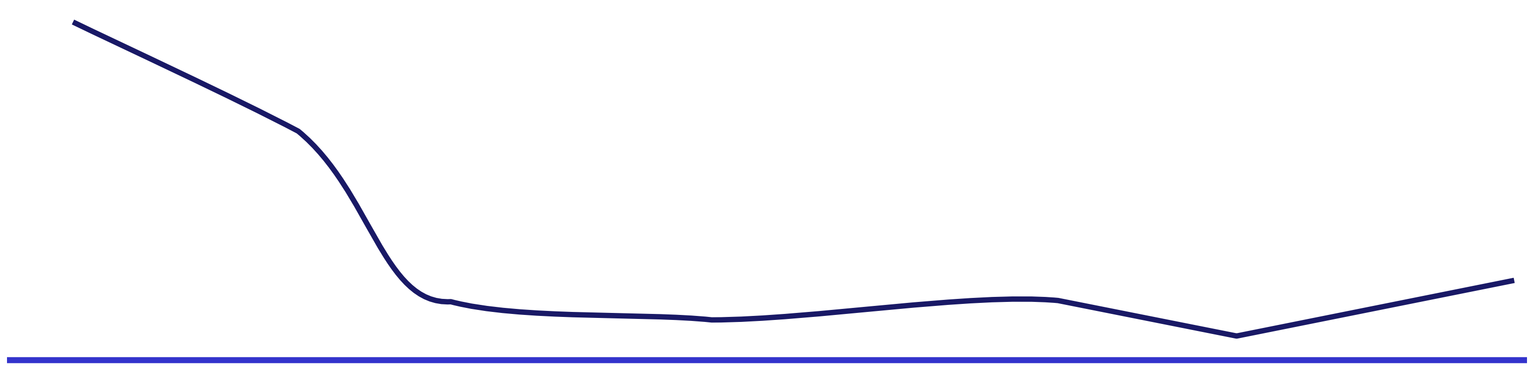
Fair Cost Sharing, Smoothly Adaptive

Key Lemma #1:

So long as all $p_i \geq \epsilon$ for constant $\epsilon > 0$, whp the cost will reach $O(OPT \cdot \log(mn))$ within poly(n) steps.

Final step for adaptive model:

If *many players of each type* can show that once cost is low, it will *never* get high again.



Fair Cost Sharing, Smoothly Adaptive

Final step for adaptive model:

If *many players of each type* can show that once cost is low, it will *never* get high again.

Proof sketch:

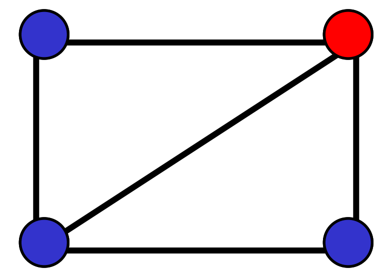
Say cost is low at time t_0.

- Hard to analyze cost of state directly, instead track upper bound $c^*(S_t) = cost(S_{t_0} \cup \ldots \cup S_t)$.
- c^* changes at most m times.
- Many players of each type $\Rightarrow$ average cost of each is low compared to c^*. Each change to c^* is small. (c^*/n_i)

Total cost ever at most: $cost(S_0)(1 + 1/n_{min})^m$

Consensus games

- Graph G, vertices have two actions: RED or BLUE.



$$\text{cost}_i(s) = \sum_{(i,j)\in E} I_{(s_i \neq s_j)}$$

Pay 1 for each edge with endpoints of different color.

$\text{cost}(s) = \sum_i \text{cost}_i(s) + 1$

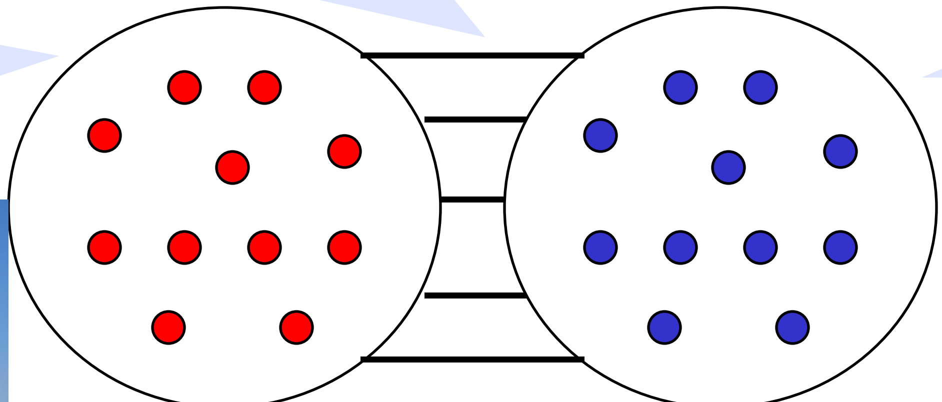
- OPT = all RED or all BLUE. Cost(OPT) = 1.

Consensus games

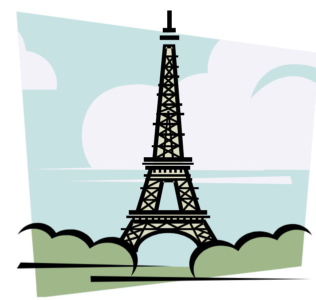
- OPT is an equilibrium so PoS = 1. But PoA = $\Omega(n^2)$.

Two cliques of size n. Each node has ϵn nbrs in other clique, $\epsilon < 1$.

English units



Metric system



- In fact, the bad equilibrium can be pretty stable.
- Even if proposal = "all BLUE", for any $p < \frac{1}{2}$, if $\epsilon < \frac{1}{2} - p$ then whp BR is to keep orig color and so no change….

Consensus games

Main result:

For any graph, any initial configuration, if $p > \frac{1}{2}$, then whp play will reach optimal in $O(n \log^2 n)$ steps. [proposal = all BLUE]

Main idea:

• Two ways a node can become blue: by choosing the proposed action or because it has more blue neighbors than red neigh, so BR is blue

• Even if many dependencies among neighbs, Pr(BR is blue) increases quickly over time.

Conclusions

Propose a novel perspective for leading dynamics to a good equilibrium and get around inherent lower bounds.

- Analyze process where some entity (who studies the game and discovers a good behavior) proposes a good joint action.
- Players don't trust, so view proposal and best-response as two "experts" and run arbitrary learning alg between them
- Positive results for cost-sharing and consensus games.

Open Questions

- Remove restriction on many players of each type for adaptive model.
- Extend model to allow multiple proposed actions, hope to do (nearly) as well as the best.
- Alternative ways to give players more info about game they are playing to allow them to reach good states fast?