The Query Complexity of Edit Distance

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Joint work with:

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Edit Distance (or Levenshtein Distance)

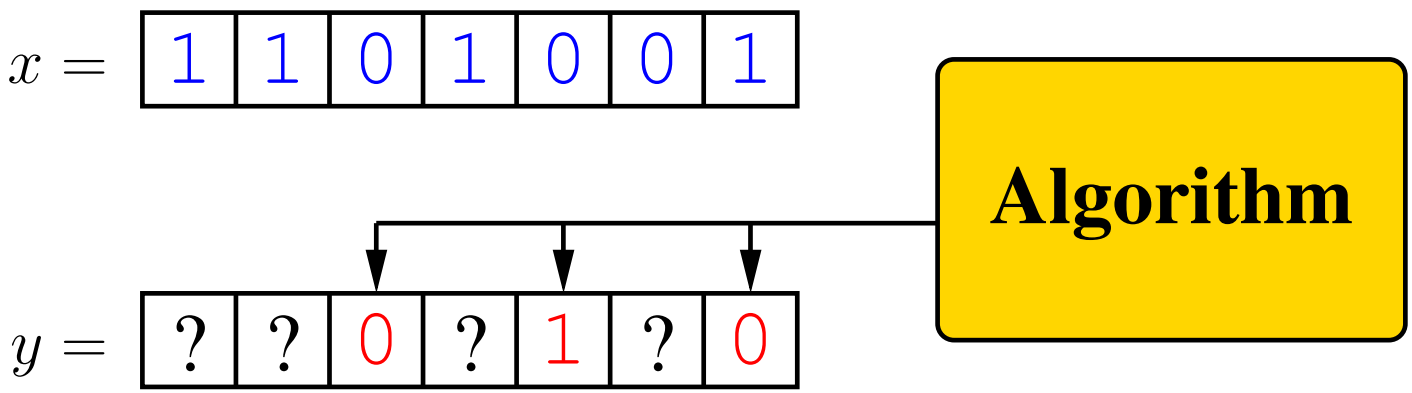
$\mathrm{ed}(x, y) =$ number of deletions, insertions, and substitutions to transform x into y

algorithm
algerithm
algebrithm
algebrathm
algebrath
algebrah
algebra

$\mathrm{ed}(\text{algorithm}, \text{algebra}) = 6$

The Model

- Input:
  - two strings x and y of length n
  - x is known to the algorithm
  - y is not known, the algorithm can query it



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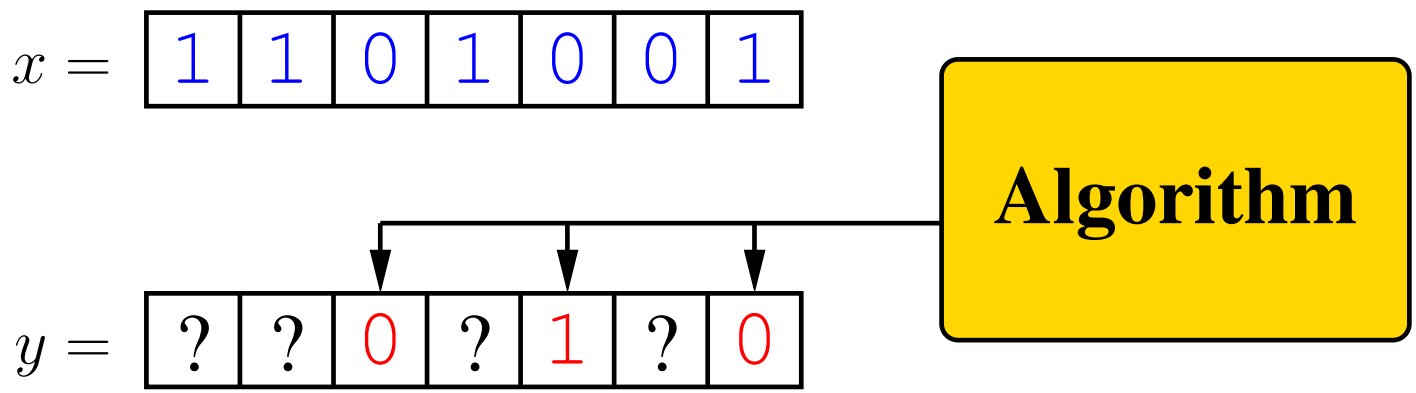
- Sample question: How many queries are necessary to tell $\mathrm{ed}(x,y) \leq .2n$ from $\mathrm{ed}(x,y) \geq .6n$?



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- Telling edit distance $O(n/\alpha)$ from $\Omega(n)$:

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Edit Distance vs. Ulam Distance

- Ulam distance:
  - $\mathrm{lcs}(x, y) =$ longest common subsequence length
  - Ulam distance defined for strings with no element appearing twice
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- First separation between the two

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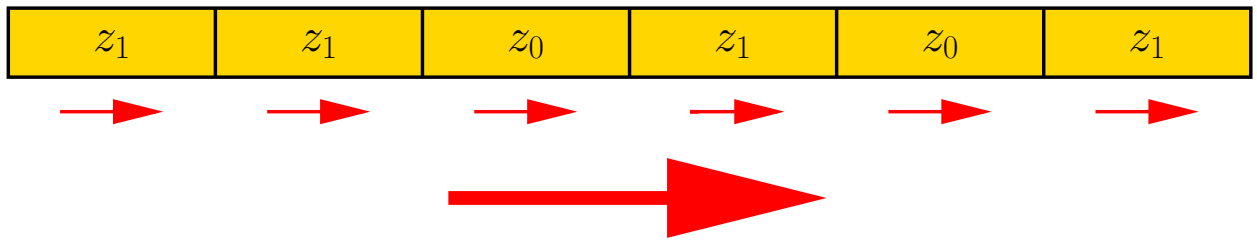

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  - Will need $\Omega(\log^2 n)$ queries

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Stay tuned!!!

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- Exact query complexity of edit distance still open

Thank you!