Comparing the strength of query types in Property Testing

The case of k-colorability

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Input graphs

Graphs of general density

$G=(V,E)$, $|V|=n$, $|E|=m$

$d := 2m/n$ – average degree of G

$O(1) \le d(n) \le O(n)$ – no apriori assumptions

Measuring distance: $\mathrm{dist}(G,H)=\mathrm{edit.dist}(G,H)/|E(G)|$

P – property: $\mathrm{dist}(G,P) := \min\{\mathrm{dist}(G,H): H \in P\}$

w.r.t. the actual size of G

- PR'02; KKR'04

Query types

- Designed to accommodate various graph densities

- degree queries: what is $d(v)$?
  - mostly convenience

- pair queries: whether $(u,v) \in E(G)$?
  - like in dense graphs/adjacency matrix

- neighbor queries: who is the i-th neighbor of v?
  - like in bounded degree graphs/incidence lists
  - typically a random neighbor of v suffices

New query type – group query

Group query: $v \in V$, $S \subset V$

Q: whether *G* has an edge between *v* and *S*?

- Motivated by group testing
- Stronger than pair query ($S=\{u\}$)
- Can be used to emulate a neighbor query in $O(\log n)$ group queries

  $\Rightarrow$ Essentially at least as strong as pair+neighbor queries combined
- Can recover all $d(v,S)$ edges between *v* and *S* in $O(d(v,S)\cdot\log|S|)$ queries

Main task

- Comparing strength of various query models

- only pair queiries
- only neighbor queries
- pair+neighbor combined
- group queries

Test case studied: *P* := *k*-colorability, *k*≥3 constant

($k \geq 2$)

Results

	Pair Queries	Neighbor Queries	Pair&Neighbor Queries	Group Queries
Upper Bound	$\tilde{O}((\frac{n}{d})^2)$	$O(n)$	$\min\{\tilde{O}((\frac{n}{d})^2), O(n)\}$	$\tilde{O}(\frac{n}{d})$
Lower Bound	$\Omega((\frac{n}{d})^2)$	$\Omega(n^{1-\frac{1}{\lceil (k+1)/2 \rceil}})$ $\Omega(n \cdot d^{-\frac{1}{\lceil k/2 \rceil - 1}})$ if $k \geq 6$	$\Omega(\frac{n}{d})$ also for 2-sided error	$\Omega(\frac{n}{d})$ also for 2-sided error

Table 1: Results for one-sided error testing of k-colorability

Interpretation, conclusions

- Group queries are at least as strong as the combined model; sometimes are strictly stronger (for *k*=2, cf. KKR'04);
- Neighbor queries are better suited for sparse graphs; increasing density does not necessarily make it easier;
- Pair queries are better suited for dense graphs;
- Combined model (pair+neighbor queries) is strictly stronger than best(pair,neighbor), at least for some problems.

Thank you for your (short…) attention!