Testing Euclidean Spanners

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Testing geometric graphs

- G=(P,E)
- P is a point set in $\mathcal{R}^d$
- G may be undirected or directed
- Bounded degree graph model + constant time access to point coordinates
- Algorithm has 1-sided error

Definition (Euclidean spanner)

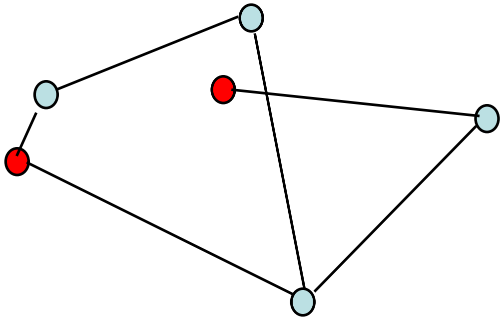
Geometric graph G=(P,E) is called (1+δ)-spanner, if for every pair of points $p,q \in P$:

$$D_G(p,q) \le (1+\delta)\ ||p-q||_2$$

Question:

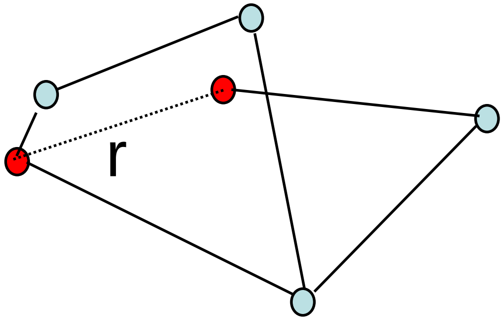
- Can we test Euclidean spanners?

Witness for rejection



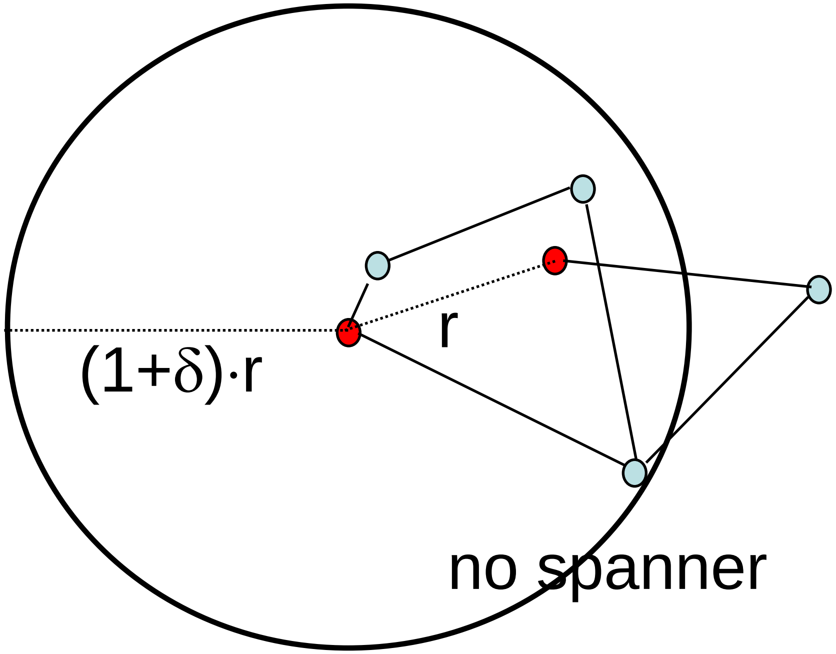
no spanner

Witness for rejection

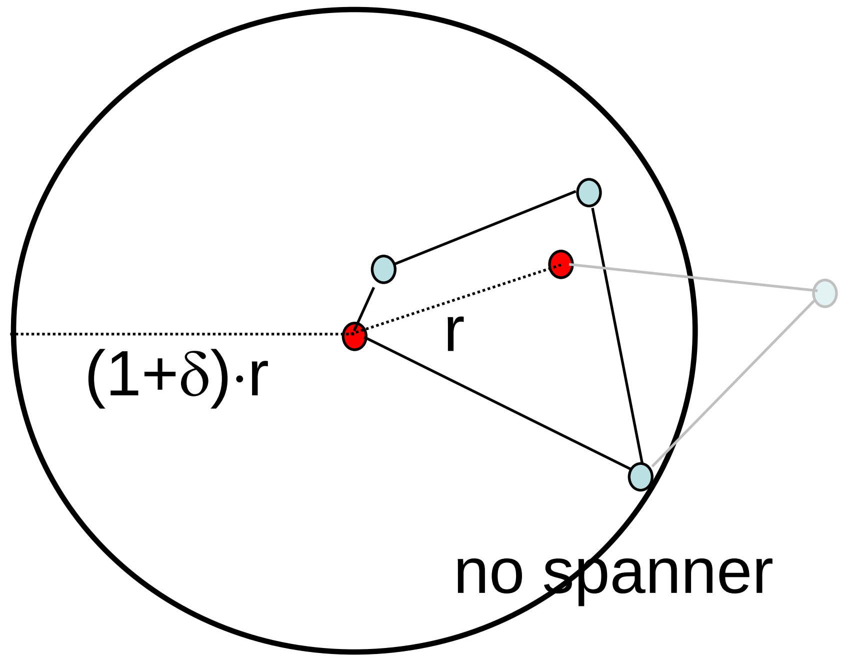


no spanner

Witness for rejection

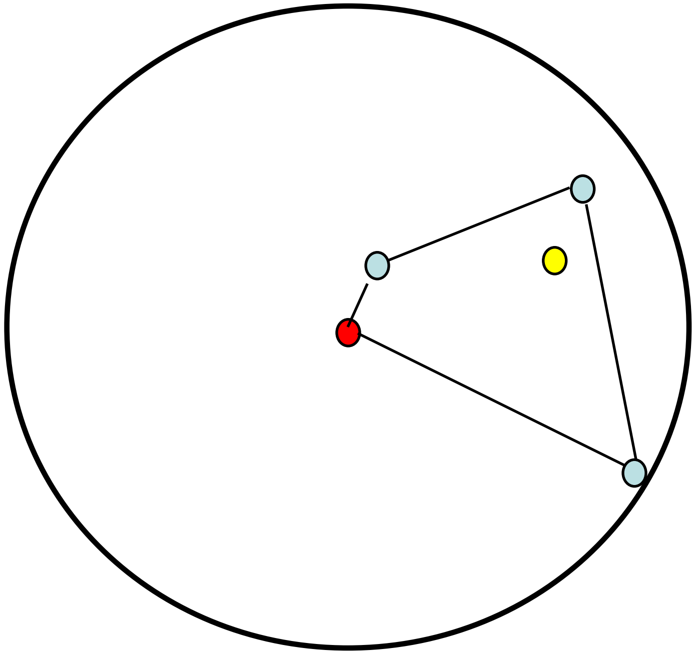


Witness for rejection



Rejection Witness

- Graph induced by disc around a point p
- A point in this disc with too long distance to p

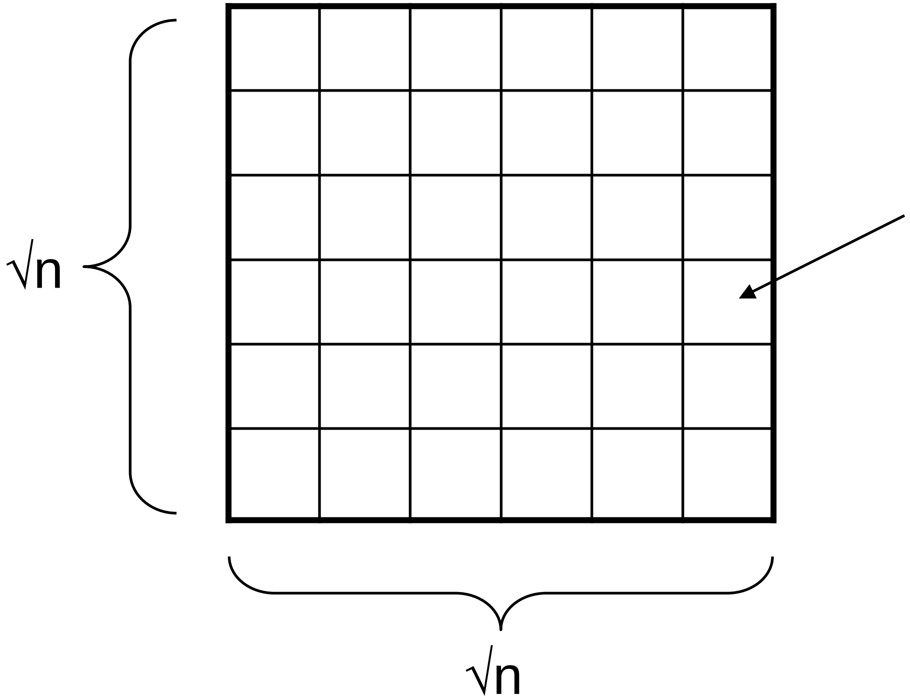


SpannerTest(G,ε,δ)

1. Sample set S of s points uniformly at random
2. **for each** $p \in S$ **do**
3. perform Dijkstra‘s algorithm starting from p until k points have been visited
4. Sample set T of t points uniformly at random
5. Check, whether any pair $(p,q) \in S \times T$ is a rejection witness

Simplifying assumption

- Points are ‚uniformly spread'



Each cell contains
O(1) points

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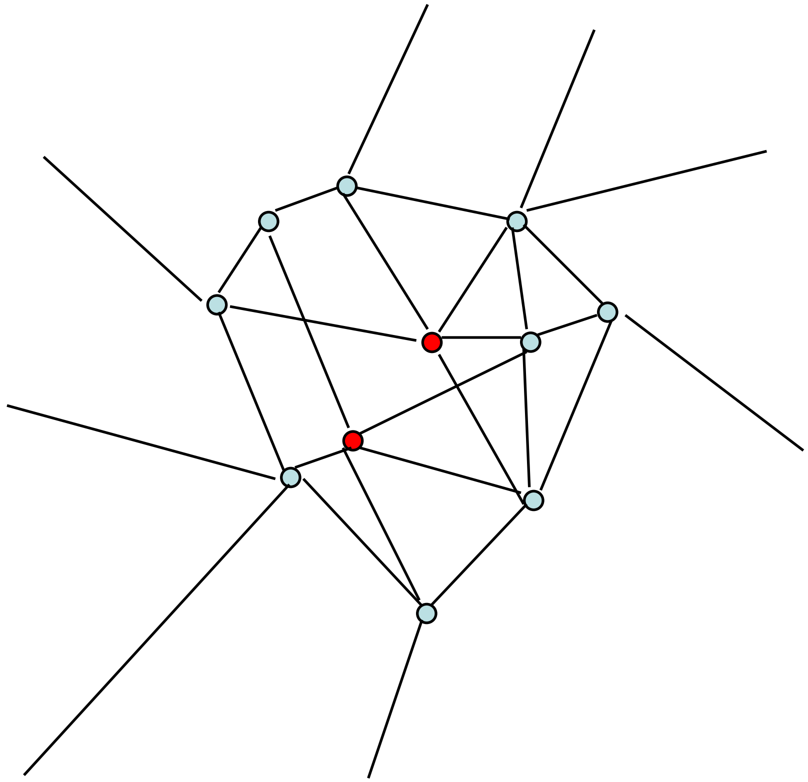
Hubs

- If the graph is locally a spanner, then it is ε-close to a spanner
- ε-far => $\Omega(\varepsilon n)$ local spanner edges are missing
- Edge (p,q) is local, if ||p-q|| is at most poly(log n, $1/\varepsilon$) cells

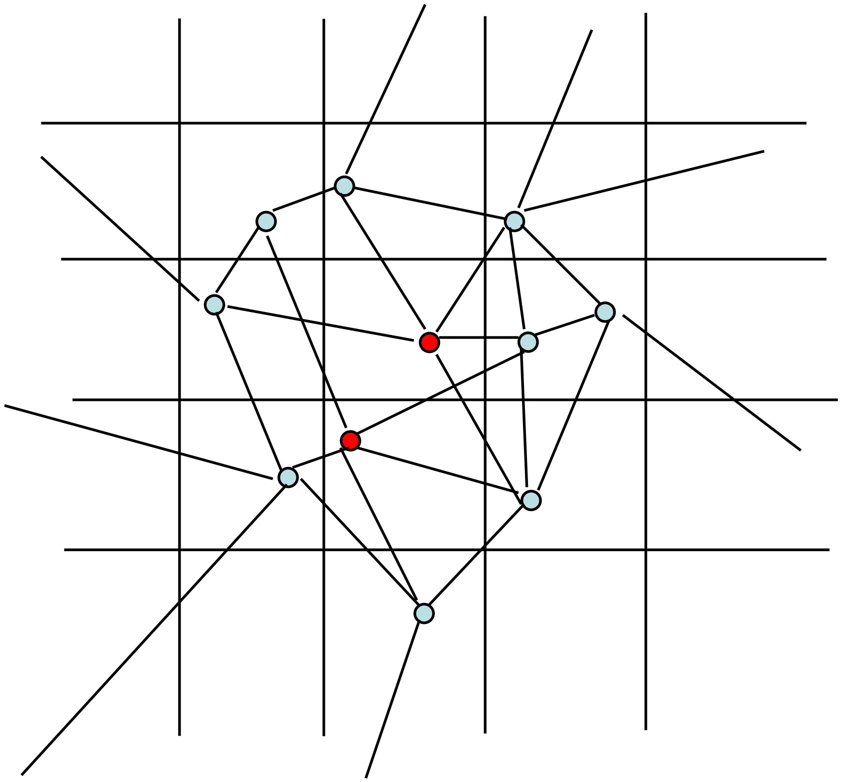
Uniform case

- Consider a ‚missing' local edge



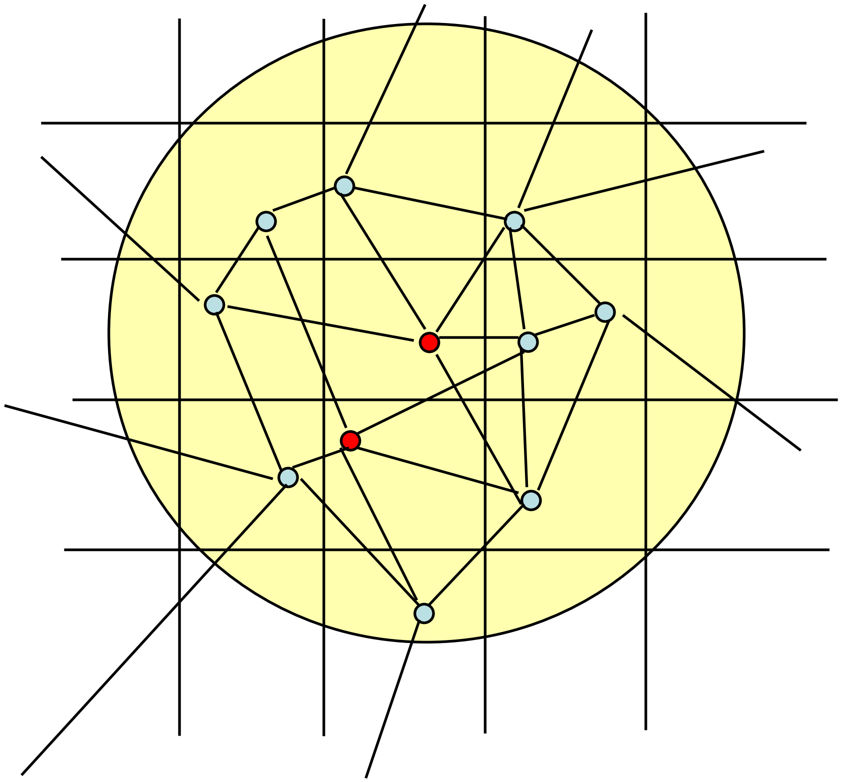
Uniform case

- Since the edge is local, it does not cross many cells



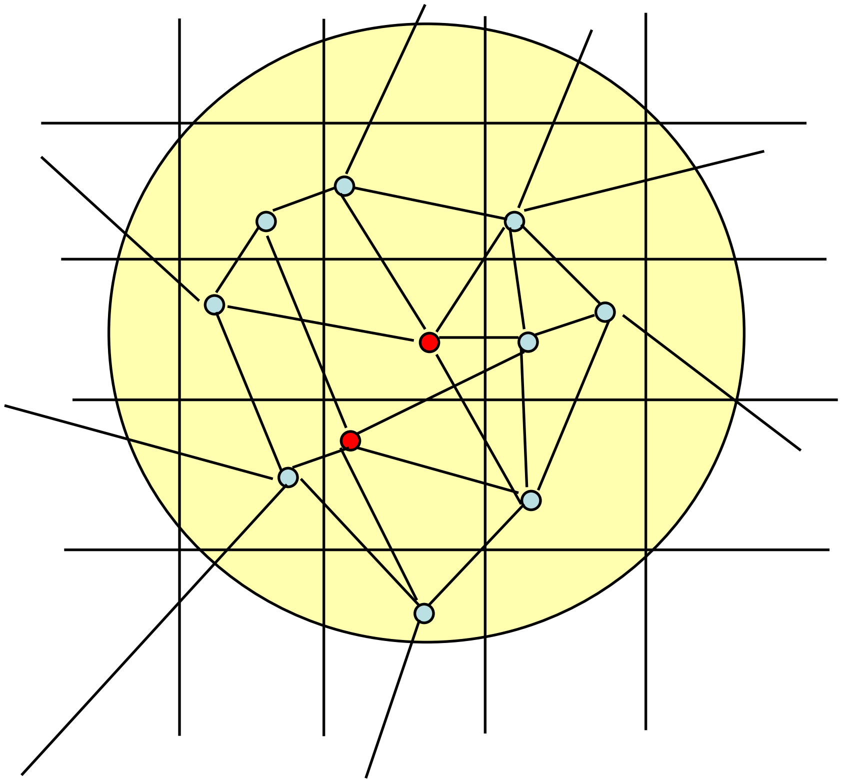
Uniform case

- Therefore, disc around endpoint is ‚large enough‘



Uniform case

- We need one end point in S and one in T $\Rightarrow$ birthday problem



Theorem*

Let P be a set of points from grid $\{1,\ldots,\Delta\}^2$. There is a property tester that tests whether a graph G=(P,E) is a $(1+\delta)$-spanner in time $O(\sqrt{n} \cdot \mathrm{poly}(\log \Delta, 1/\varepsilon, 1/\delta))$.

* Degree bound large enough and violation of degree bound in definition of ε-far is allowed

Thank you!