Prospects for Superconducting Qubits

David DiVincenzo

24.01.2013

QIP Beijing

Masters/PhD/postdocs available!
http://www.physik.rwth-aachen.de/
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(G: IQI Aachen)

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Prospects for Superconducting Qubits

Outline

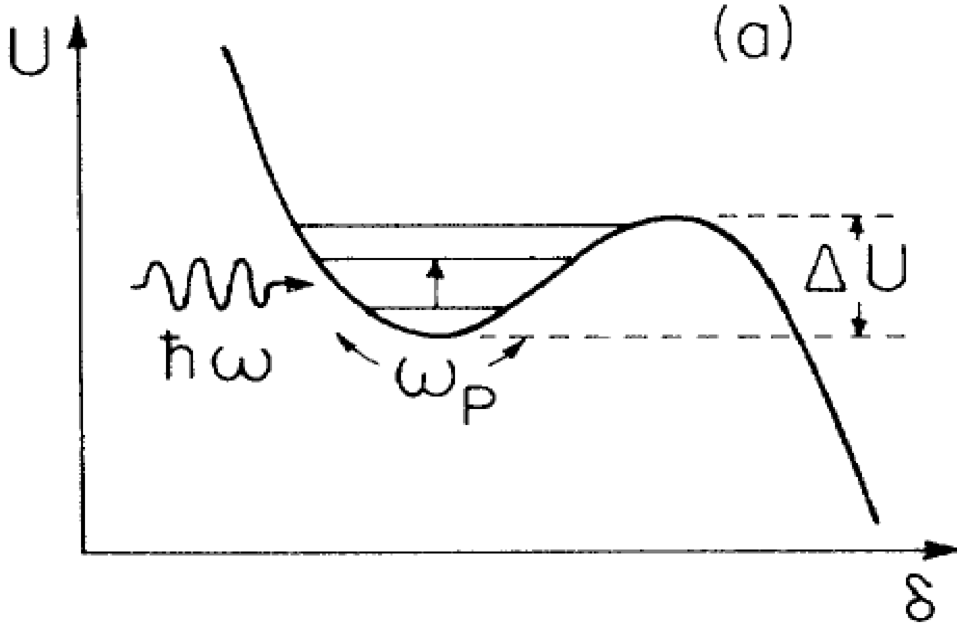
- Basic physics of Josephson devices
- A short history of quantum effects in electric circuits
- A Moore's law for quantum coherence
- Approaching fault tolerant fidelities (95%)
- Scaling up with cavities – towards a surface code architecture
- Will it work??

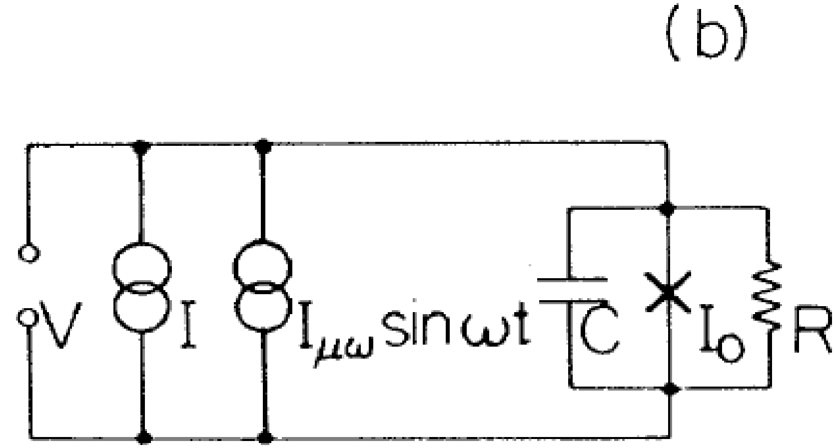
"In a machine such as this there are very many other problems due to imperfections. . . . At least some of these problems can be remedied in the usual way by techniques such as error correcting codes . . . But until we find a specific implementation for this computer, I do not know how to proceed to analyze these effects."

R.P. Feynman
"Quantum Mechanical Computers"
Optics News, February 1985

Energy-Level Quantization in the Zero-Voltage State of a Current-Biased Josephson Junction

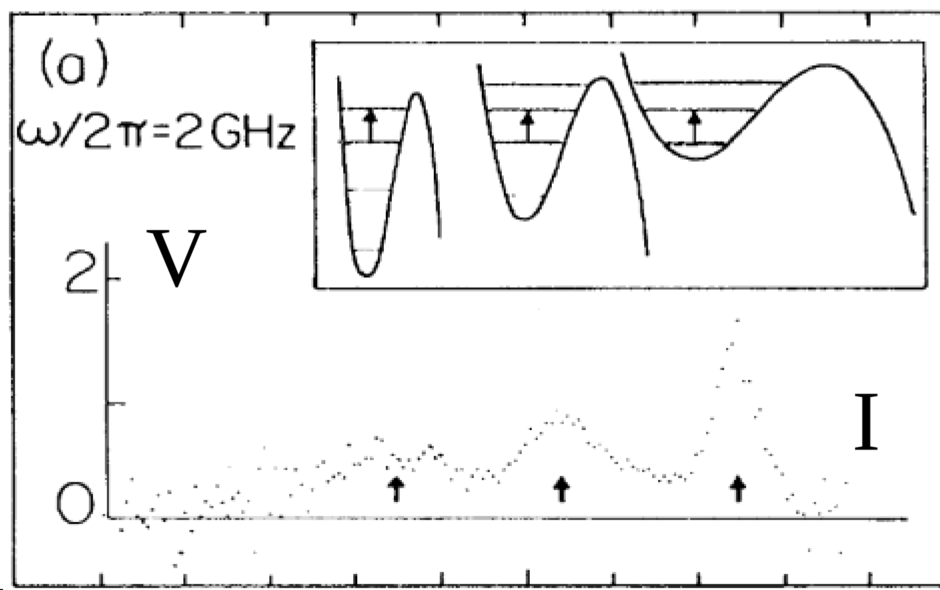
John M. Martinis, Michel H. Devoret,[a] and John Clarke





Interesting claim:

Go direct from lumped electric circuit
To Schrodinger equation

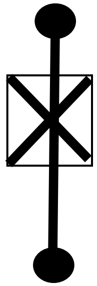


The discovery of tunnelling supercurrents*

B. D. Josephson

Cavendish Laboratory, Cambridge, England

Reviews of Modern Physics, Vol. 46, No. 2, April 1974



Current controlled by $f \propto \int V\,dt$

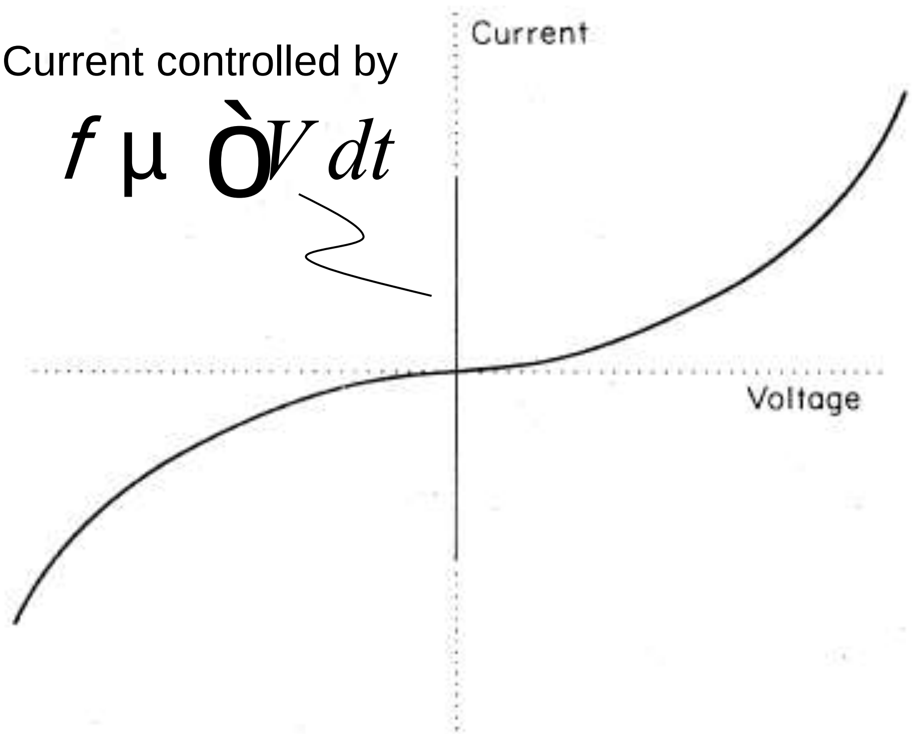


FIG. 2. Predicted two-part current-voltage characteristic of a superconducting tunnel junction.

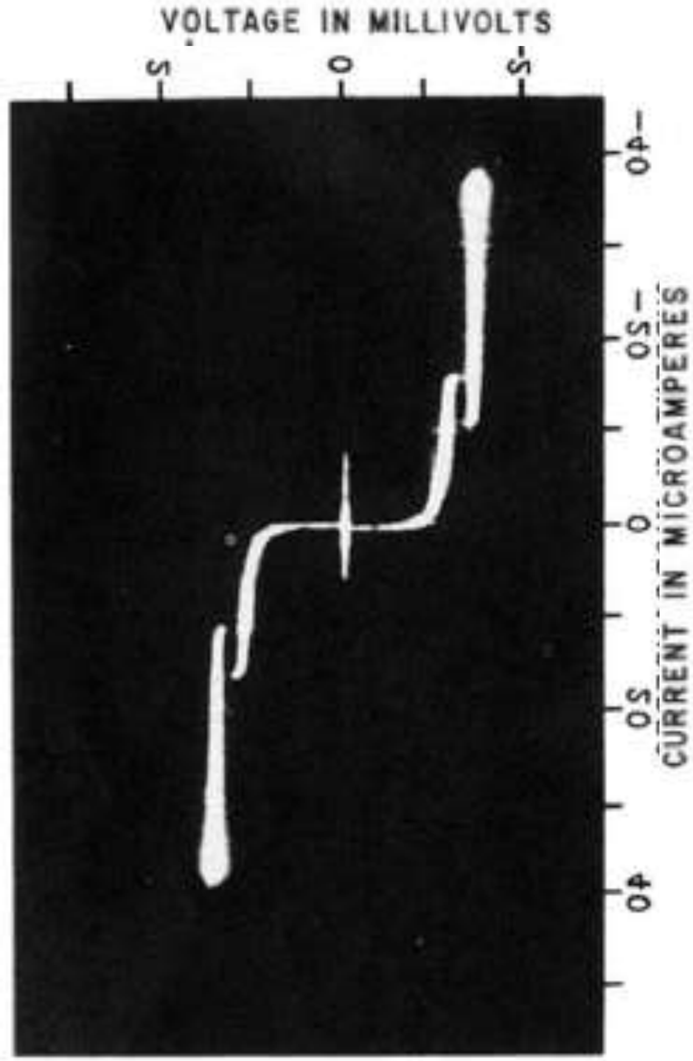


FIG. 3 The first published observation of tunnelling between two evaporated-film superconductors (Smith *et al.*, 1961). A zero-voltage supercurrent is clearly visible. It was not until the experiments of Anderson and Rowell (1963) that such supercurrents could be definitely ascribed to the tunnelling process.

The discovery of tunnelling supercurrents*

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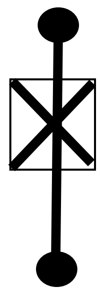


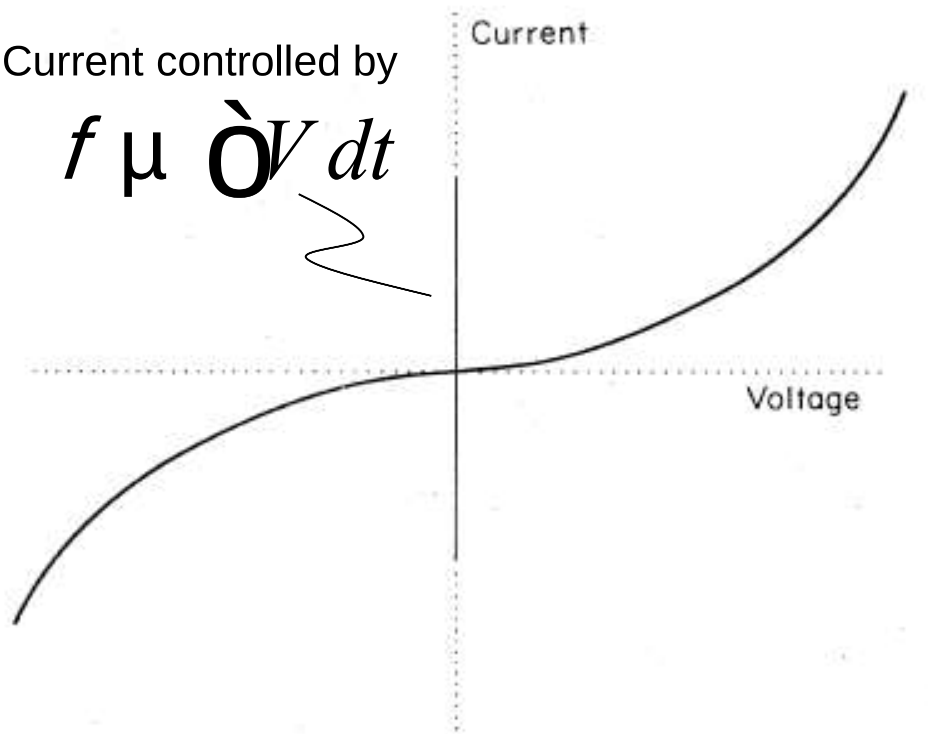
Some other history:

IBM had a large project (1975-85) to make a Josephson junction digital computer. “0” was the zero-voltage state, “1” was the finite-voltage state.

Very different from a quantum computer.

Current controlled by

$$\phi \propto \int V \, dt$$



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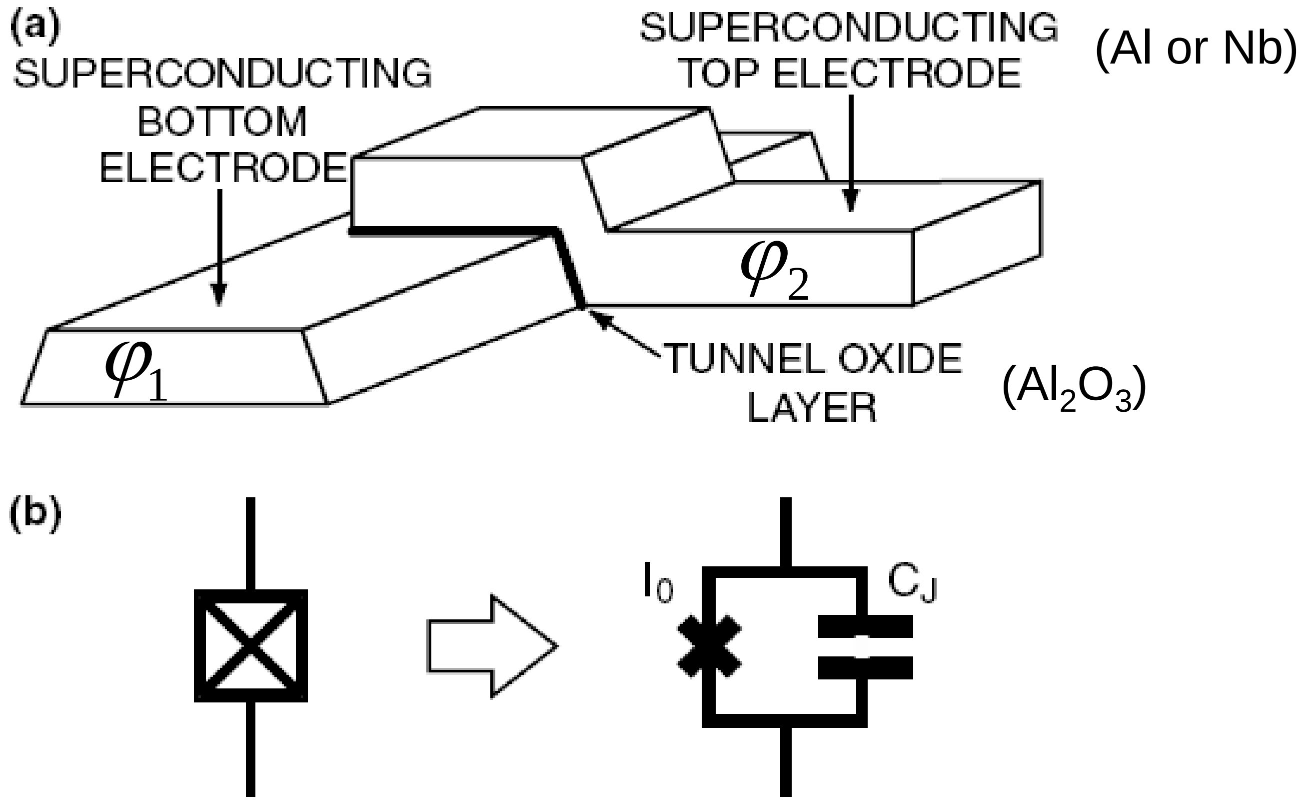


Fig. 1. (a) Josephson tunnel junction made with two superconducting thin films; (b) Schematic representation of a Josephson tunnel junction. The irreducible Josephson element is represented by a cross.

$$I = I_c \sin(\varphi_1 - \varphi_2)$$

QUANTUM FLUCTUATIONS IN ELECTRICAL CIRCUITS

S. Reynaud, E. Giacobino and J. Zinn-Justin, eds.
Les Houches, Session LXIII, 1995
Fluctuations Quantiques
Quantum Fluctuations

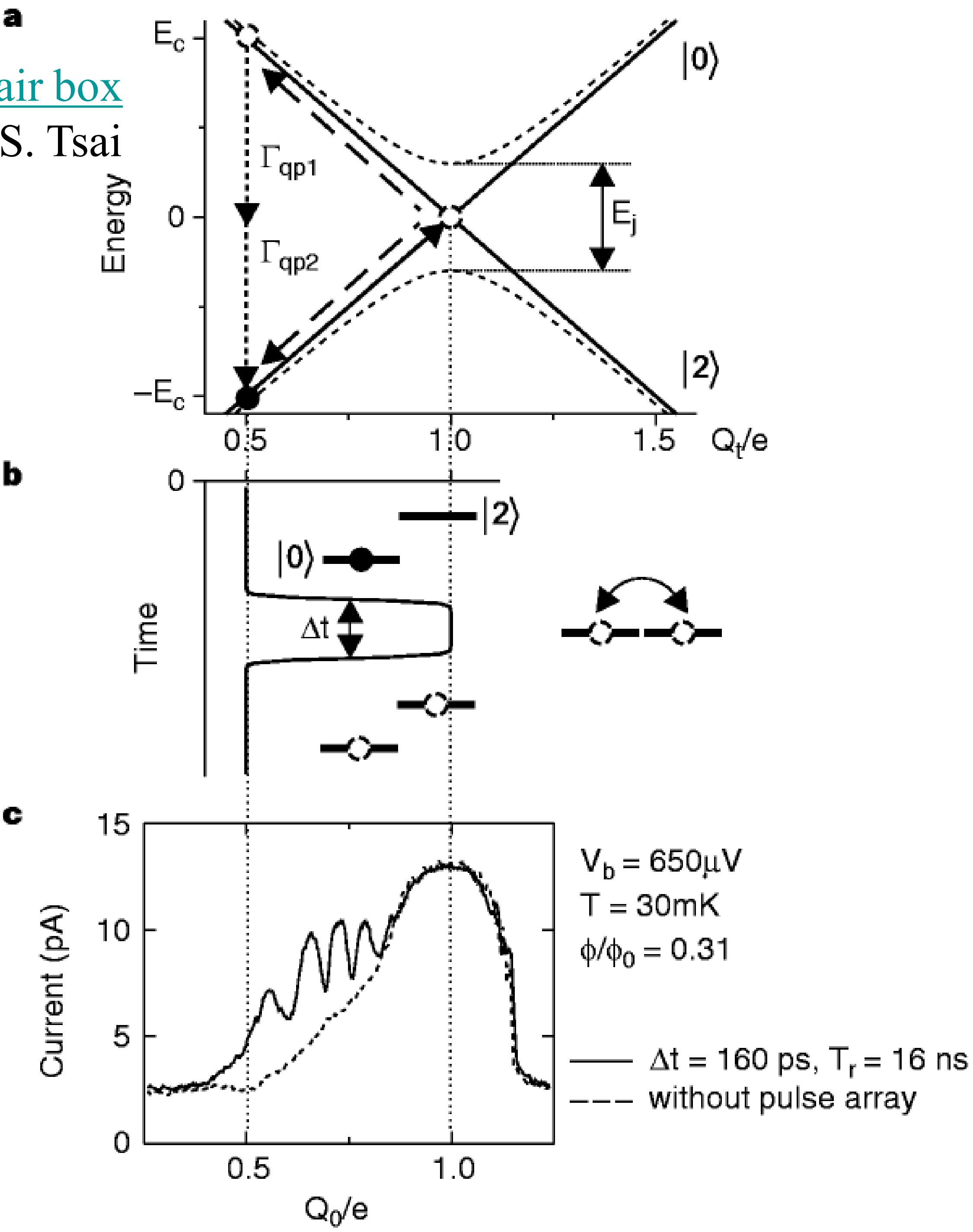
Michel H. Devoret

...er. Josephson junctions are so well coupled to their electromagnetic environment that dissipation cannot be treated as a perturbation. In fact, dissipation combines with the non-linearity of tunnel elements to produce qualitatively new quantum effects which are not encountered for example in the almost dissipation-free quantum systems studied in atomic physics. The most spectacular new quantum feature is the localization of position-like degrees of freedom when dissipation exceeds a certain threshold set by the quantum of resistance $h/(2e)^2 \simeq 6.4\ \mathrm{k}\Omega$ [8–10].

[Coherent control of macroscopic quantum states in a single-Cooper-pair box](#)

Y. Nakamura, Yu. A. Pashkin and J. S. Tsai

Nature 398, 786-788(29 April 1999)



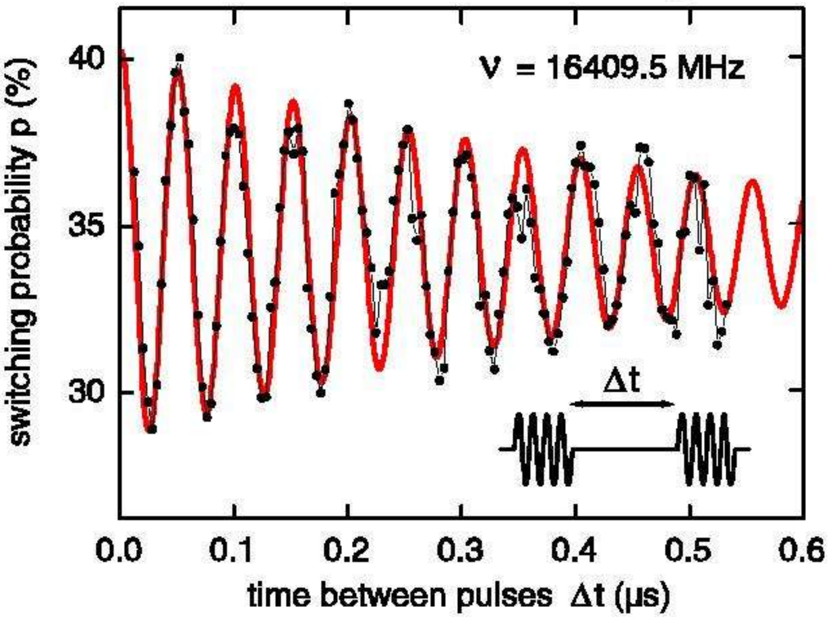
<u>Feeble</u> signs
of quantum coherence

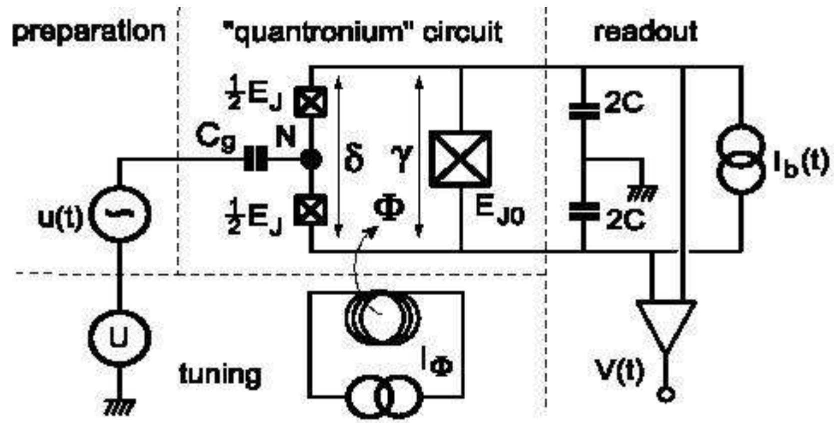
Saclay Josephson junction qubit

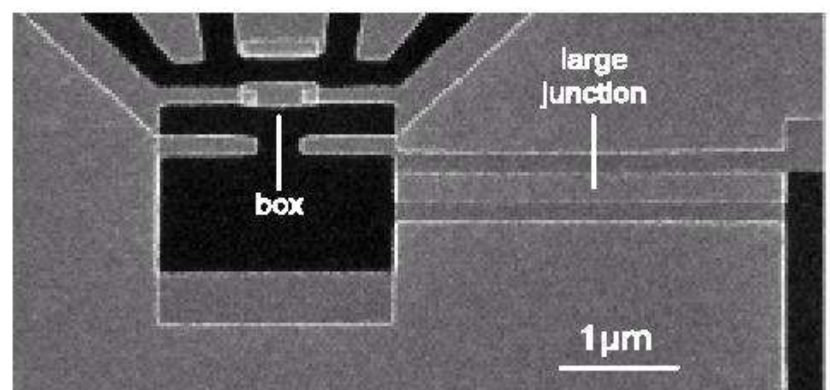
Manipulating the quantum state of an electrical circuit

Science 296, 886 (2002)

D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve and M.H. Devoret



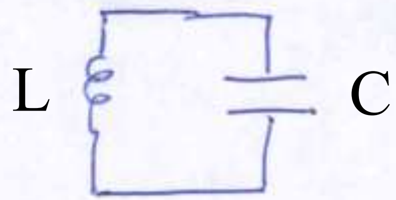




Oscillations show rotation of qubit at constant rate, with noise.

Simple electric circuit…

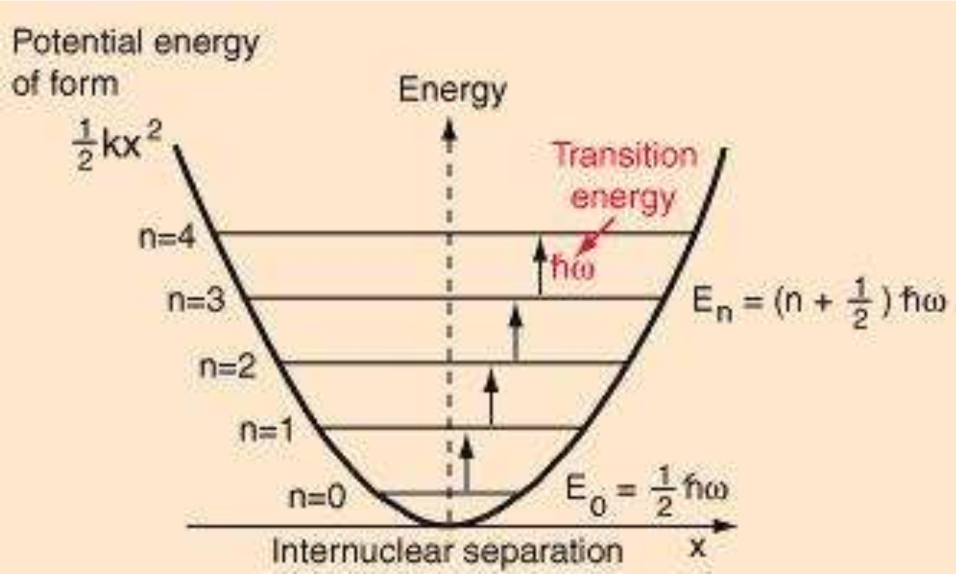
small



harmonic oscillator with resonant frequency

$$\omega_0 = 1/\sqrt{LC}$$

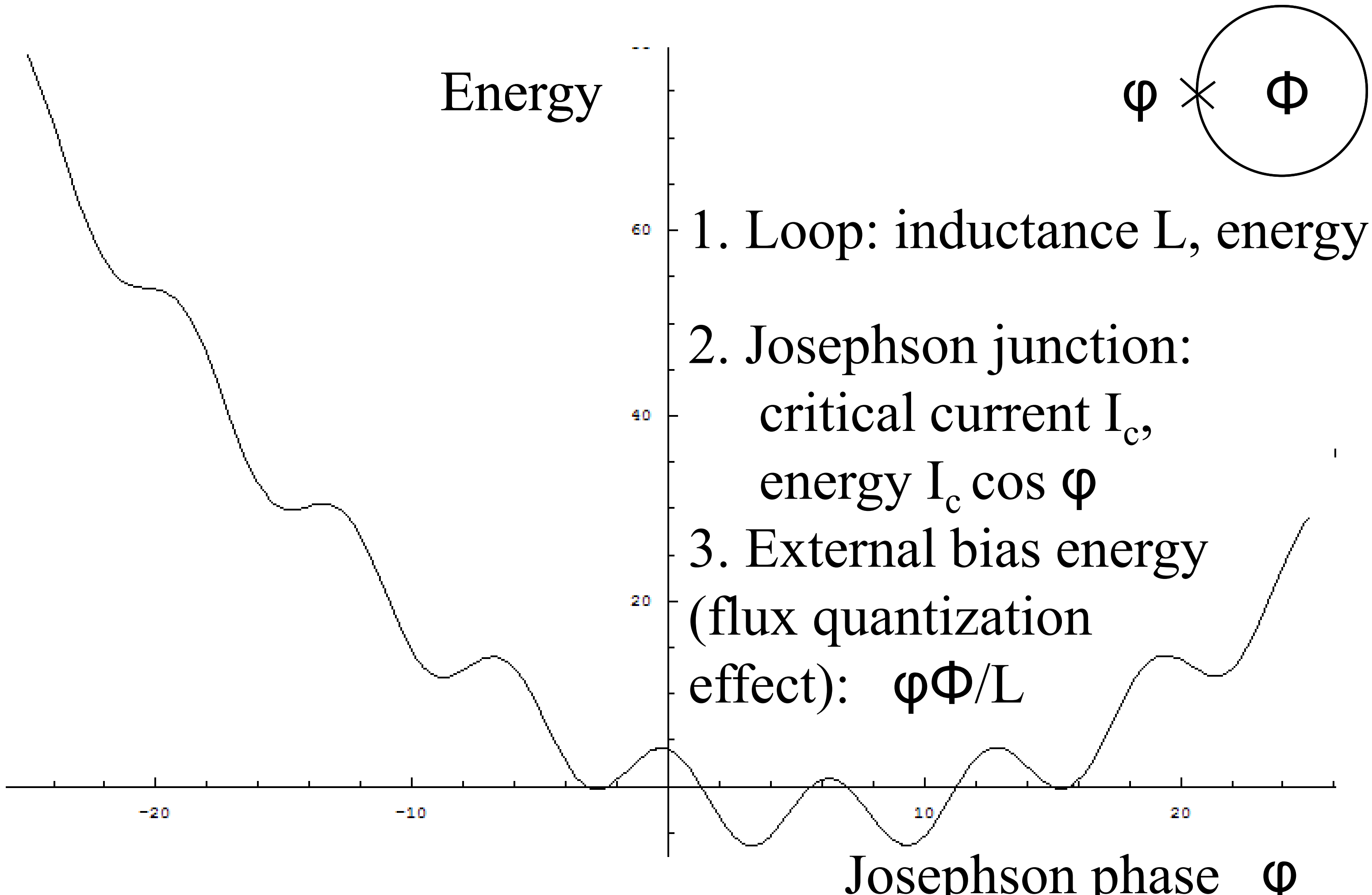
Quantum mechanically, like a kind of atom (with harmonic potential):



x is any circuit variable (capacitor charge/current/voltage, Inductor flux/current/voltage)

That is to say, it is a "macroscopic" variable that is being quantized.

Textbook (classical) SQUID characteristic: the "washboard"



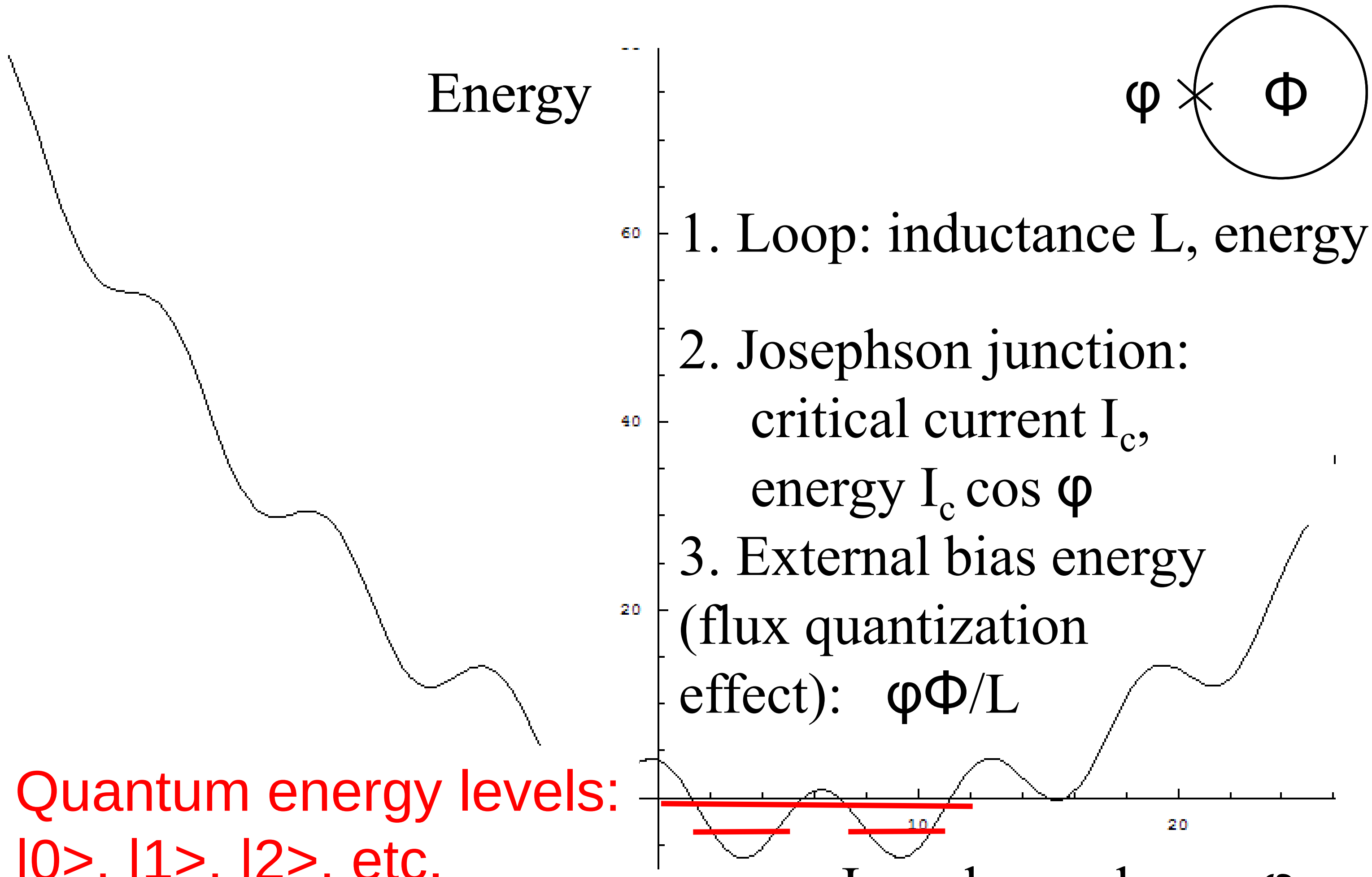
1. Loop: inductance L, energy φ^2/L

2. Josephson junction:
   critical current I_c,
   energy $I_c \cos \varphi$

3. External bias energy (flux quantization effect): $\varphi\Phi/L$

Junction capacitance C, plays role of particle mass

Quantum SQUID characteristic:



Energy

1. Loop: inductance L, energy φ^2/L

2. Josephson junction:
   critical current I_c,
   energy $I_c \cos \varphi$

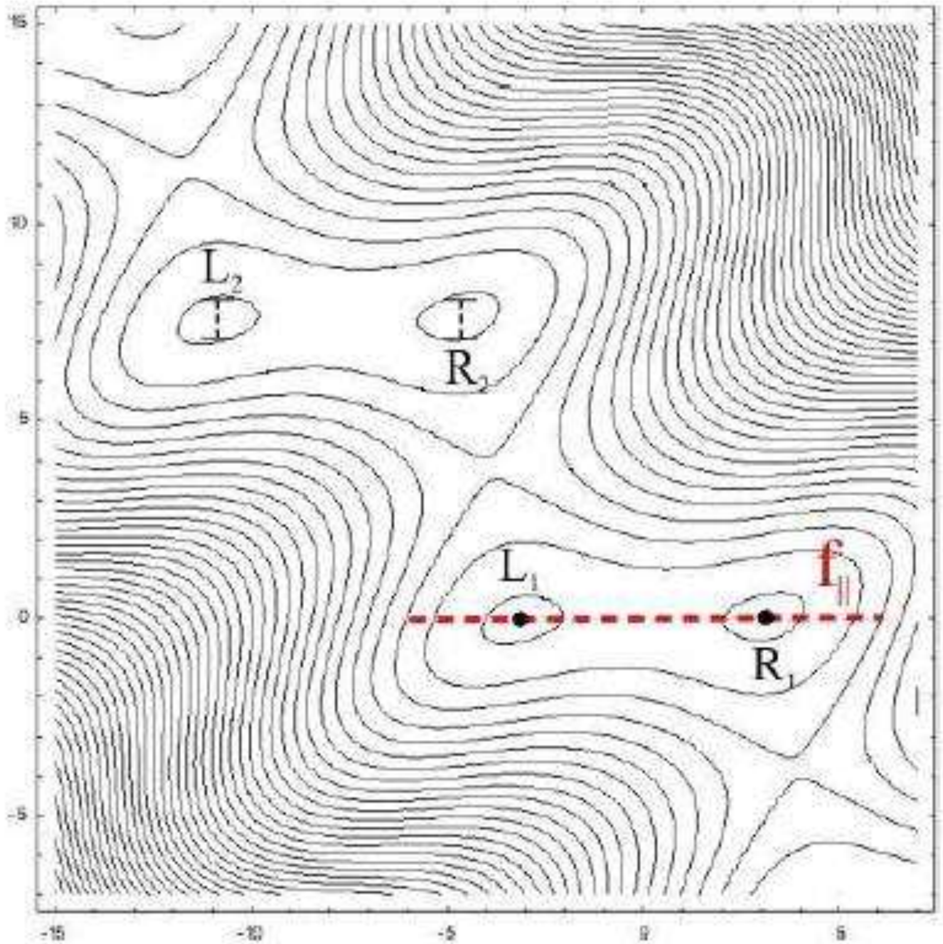
3. External bias energy (flux quantization effect): $\varphi\Phi/L$

- Quantum energy levels: |0>, |1>, |2>, etc.

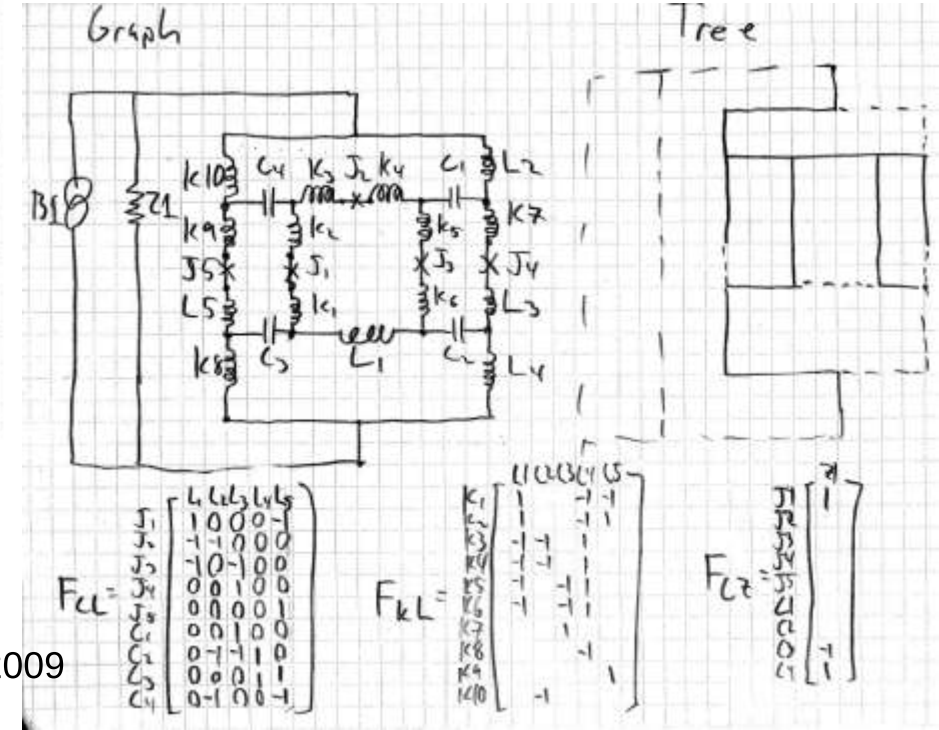
Josephson phase φ

Junction capacitance C, plays role of particle mass

Burkard, Koch, DiVincenzo, Phys. Rev. B (2004)
DiVincenzo, Brito, and Koch, Phys. Rev. B (2006).

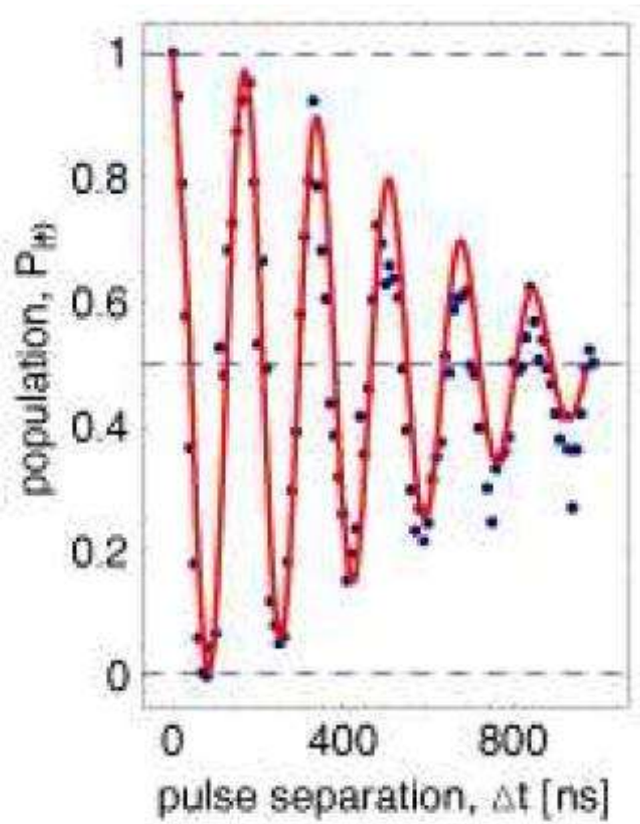


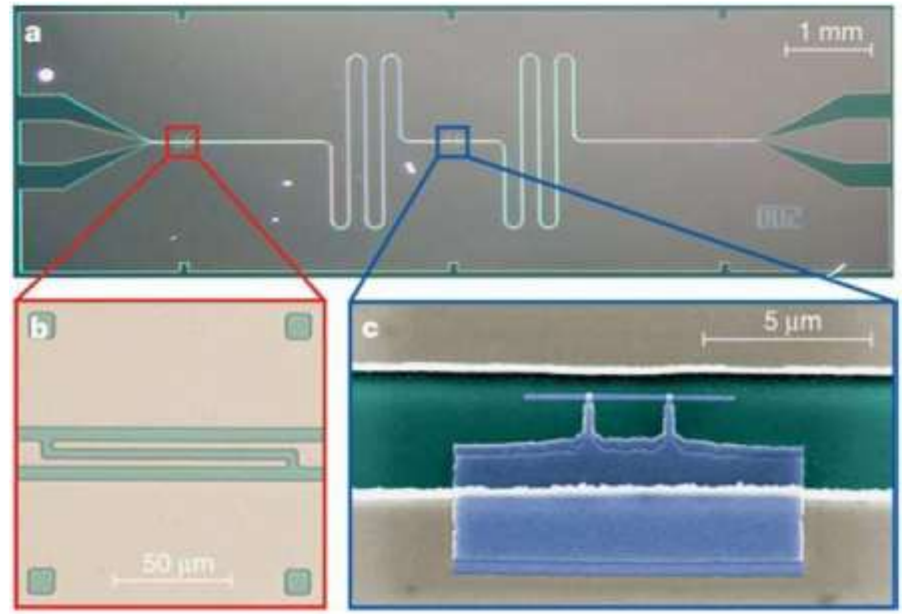
Effective potential is generally multidimensional, complex interplay
between anharmonic and harmonic parts



“Yale” Josephson junction qubit

PRL 2005;
also Nature, 2004





Approaching Unit Visibility for Control of a Superconducting Qubit with Dispersive Readout

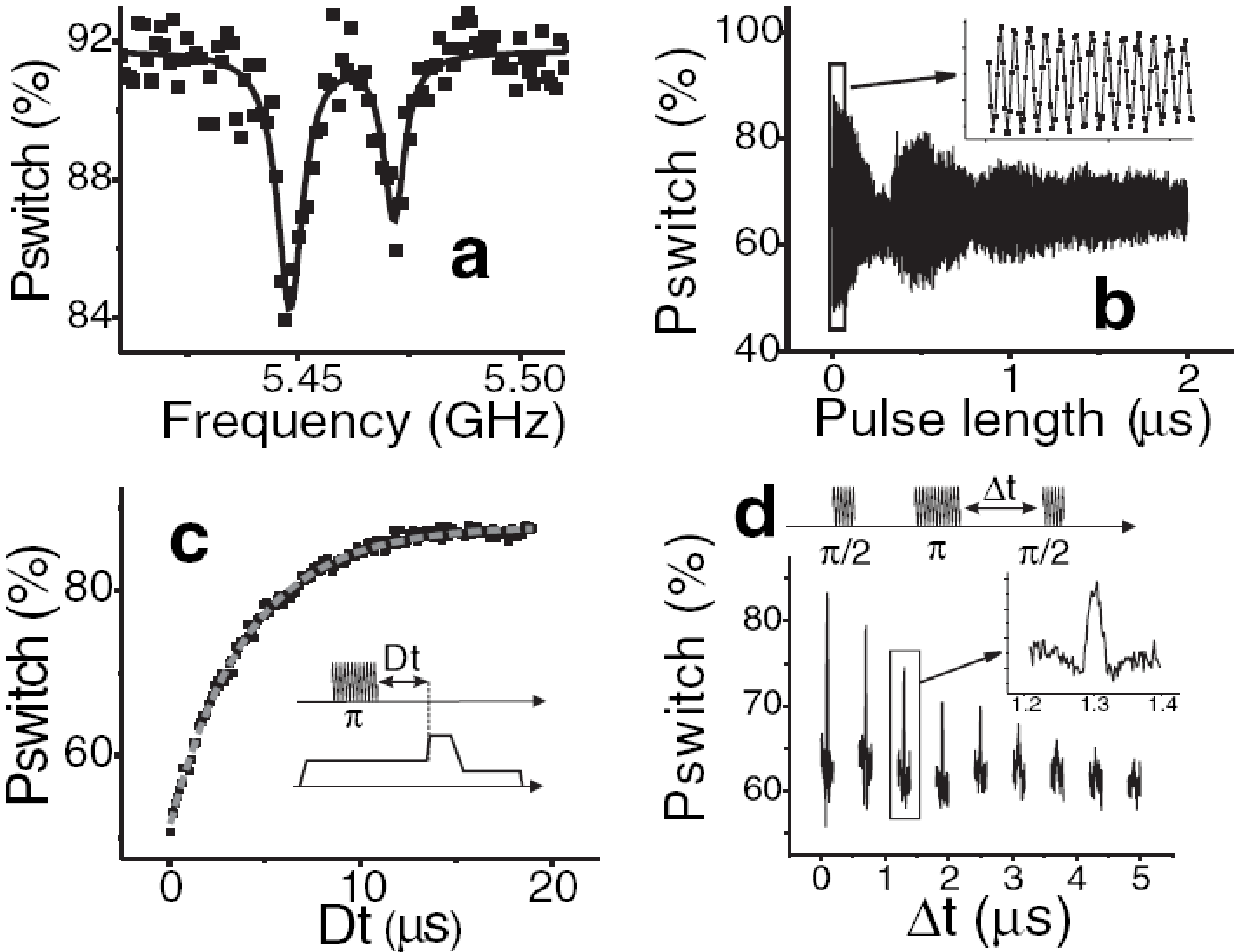
A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, J. Majer, S. M. Girvin, and R. J. Schoelkopf

arXiv:cond-mat/0502645 v1 27 Feb 2005

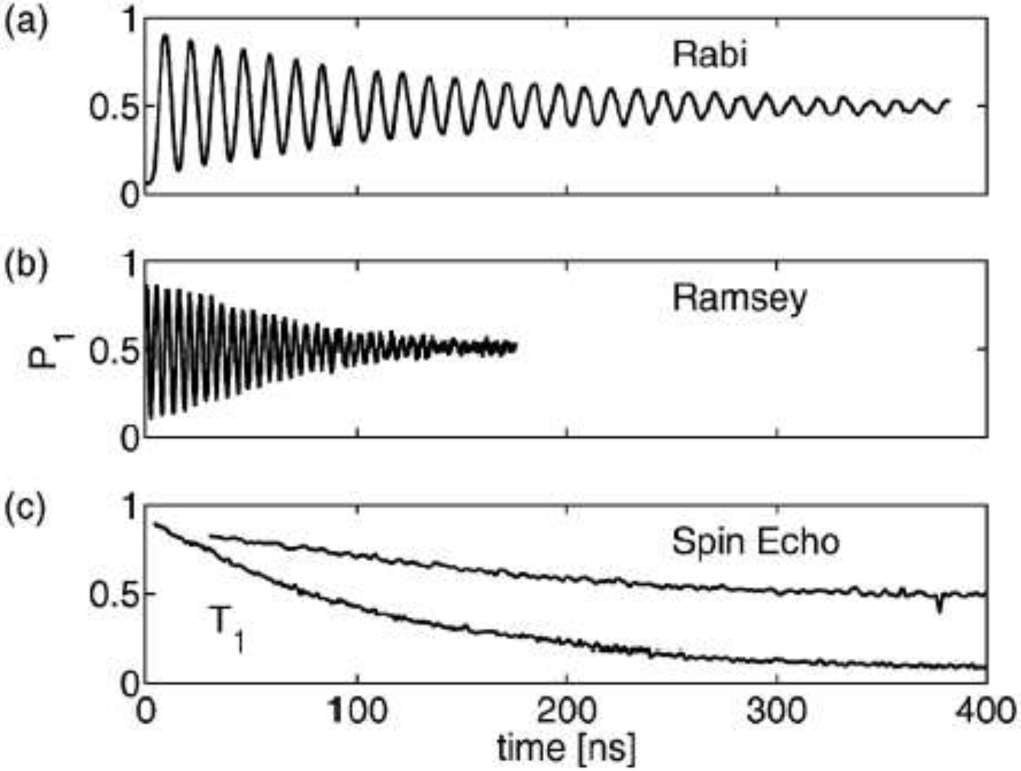
Coherence time again c. 0.5 μs (in Ramsey fringe experiment)
But fringe visibility > 90% !

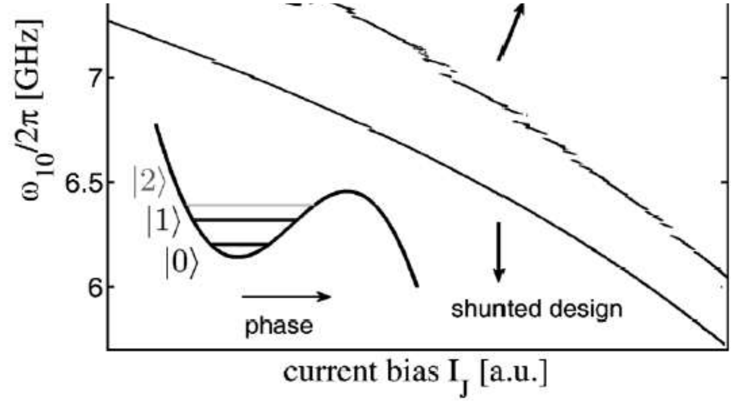
Dephasing of a Superconducting Qubit Induced by Photon Noise

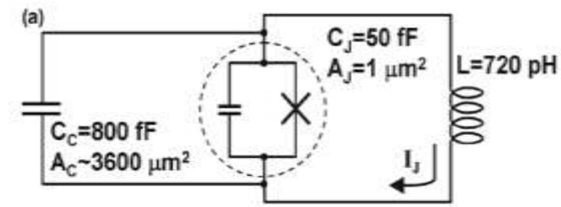
P. Bertet,[1] I. Chiorescu,[1,*] G. Burkard,[2,3] K. Semba,[1,4] C. J. P. M. Harmans,[1] D. P. DiVincenzo,[2] and J. E. Mooij[1]

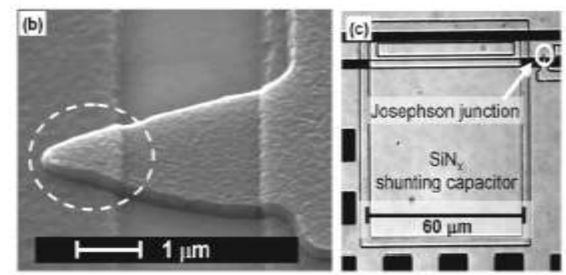


UCSB Josephson junction qubit (“phase”)









PRL **97**, 050502 (2006) PHYSICAL REVIEW LETTERS week ending 4 AUGUST 2006

State Tomography of Capacitively Shunted Phase Qubits with High Fidelity

Matthias Steffen, M. Ansmann, R. McDermott, N. Katz, Radoslaw C. Bialczak, Erik Lucero, Matthew Neeley, E. M. Weig, A. N. Cleland, and John M. Martinis*

Department of Physics and California Nanosystems Institute, University of California, Santa Barbara, California 93106, USA

Temperature dependence of coherent oscillations in Josephson phase qubits

J. Lisenfeld[1], A. Lukashenko[1], M. Ansmann[2], J. M. Martinis[2], and A. V. Ustinov[1]*
[1] *Physikalisches Institut III, Universität Erlangen-Nürnberg, D-91058 Erlangen, Germany*
[2] *Department of Physics and California Nanosystems Institute, University of California, Santa Barbara, California 93106, USA*
(Dated: February 1, 2008)

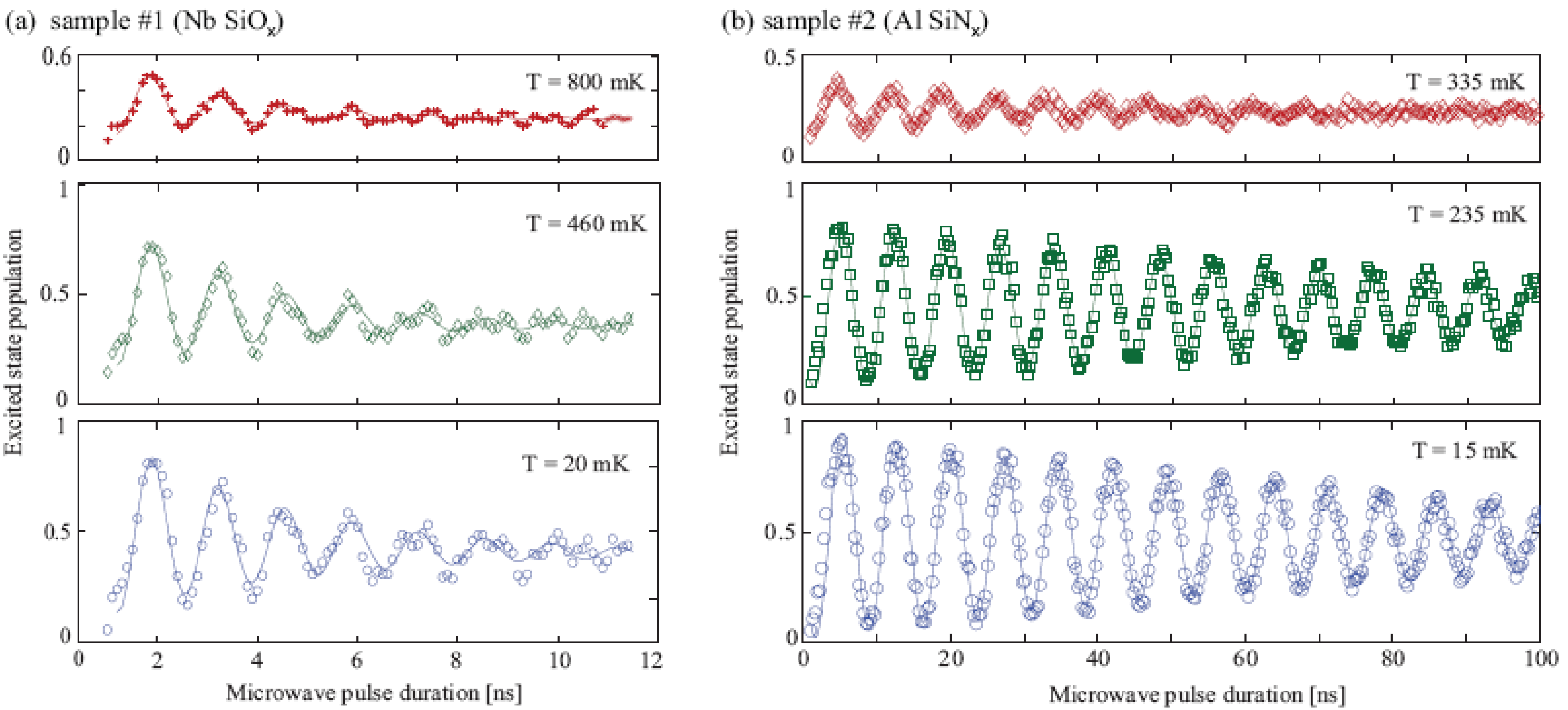
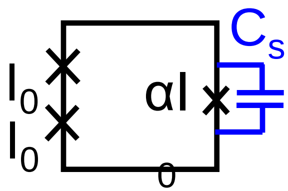
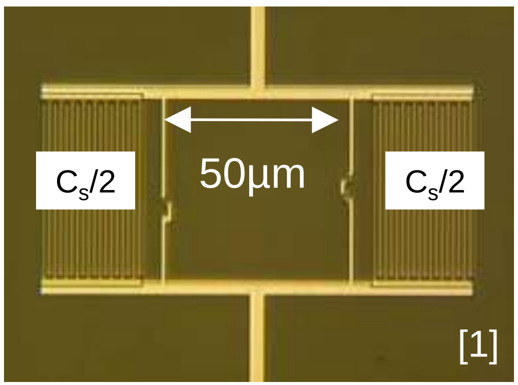


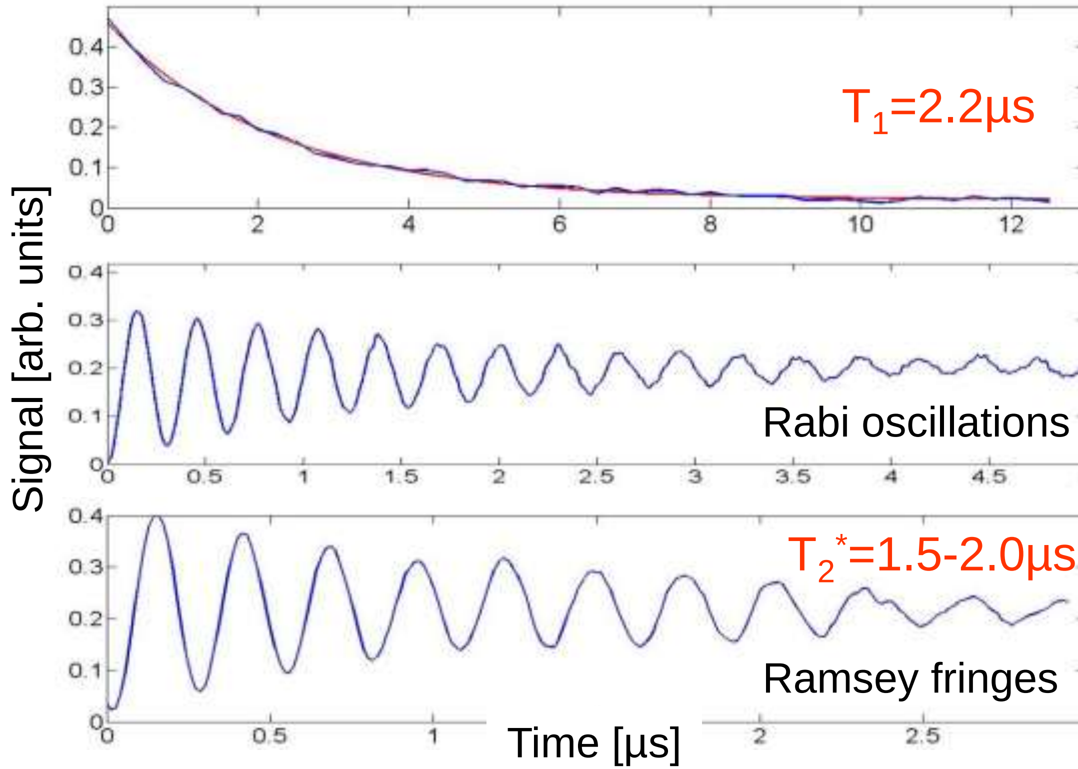
FIG. 2: (a) Rabi oscillations observed in Nb-based sample #1 with SiO_2 insulation and (b) in Al-based sample #2 featuring SiN_x insulation, at the indicated temperatures. Solid lines are a fits to exponentially decaying sine functions from which Rabi amplitude and decay time are extracted.

IBM Experiments 2010

Capacitively Shunted Flux Qubit (CSFQ)







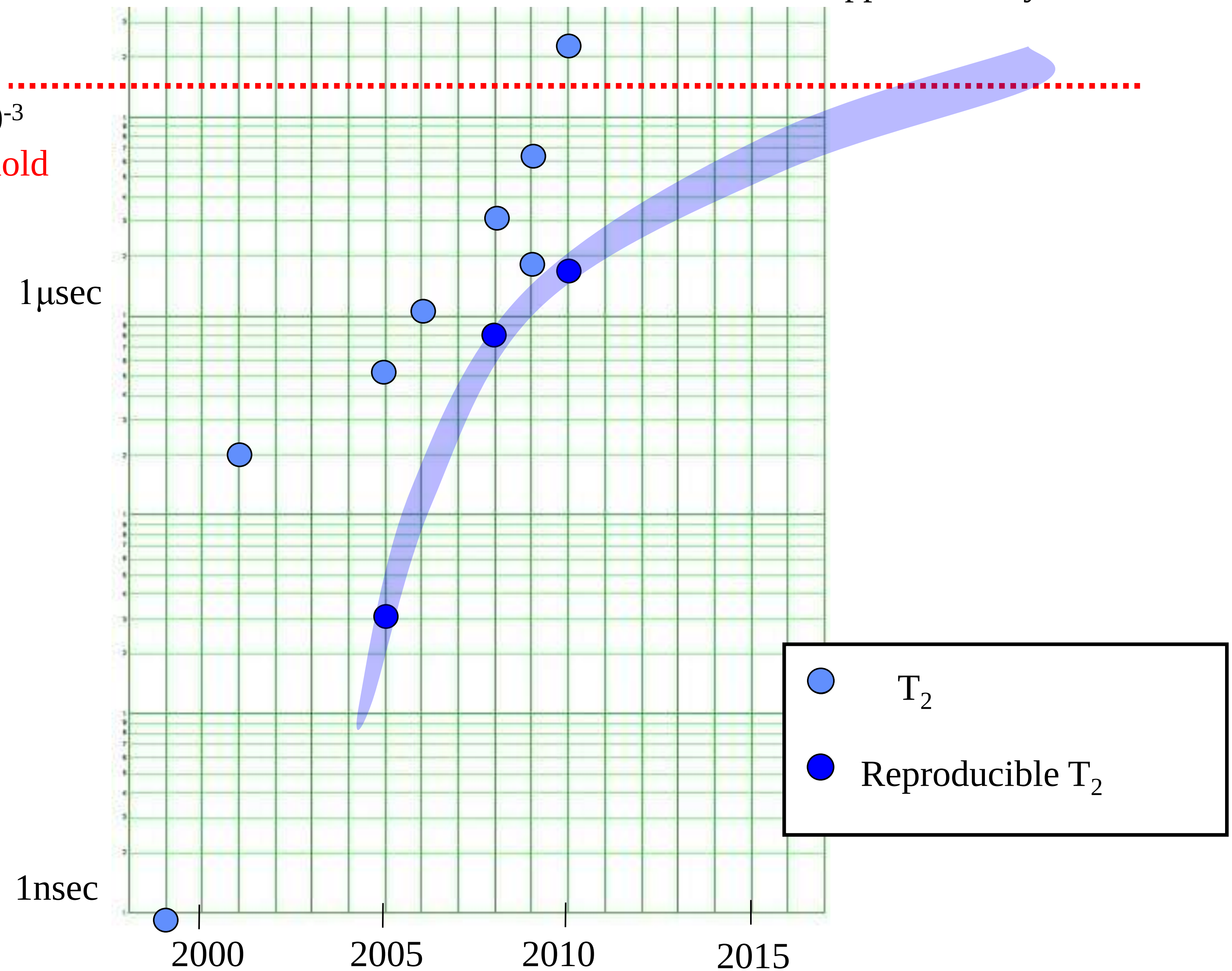
- Nine out of ten samples measured gave T1>1µs
- No device failure observed yet
- Energy loss likely dominated by dielectric surface loss

[1] M.Steffen et al., PRL, 105, 100502 (2010)

Prospects: a Moore's law for coherence time

NB: anticipated gate time approximately 30 nsec

$t_{gate}/T_2 \approx 5 \times 10^{-3}$
= noise threshold
of 2D surface
code scheme



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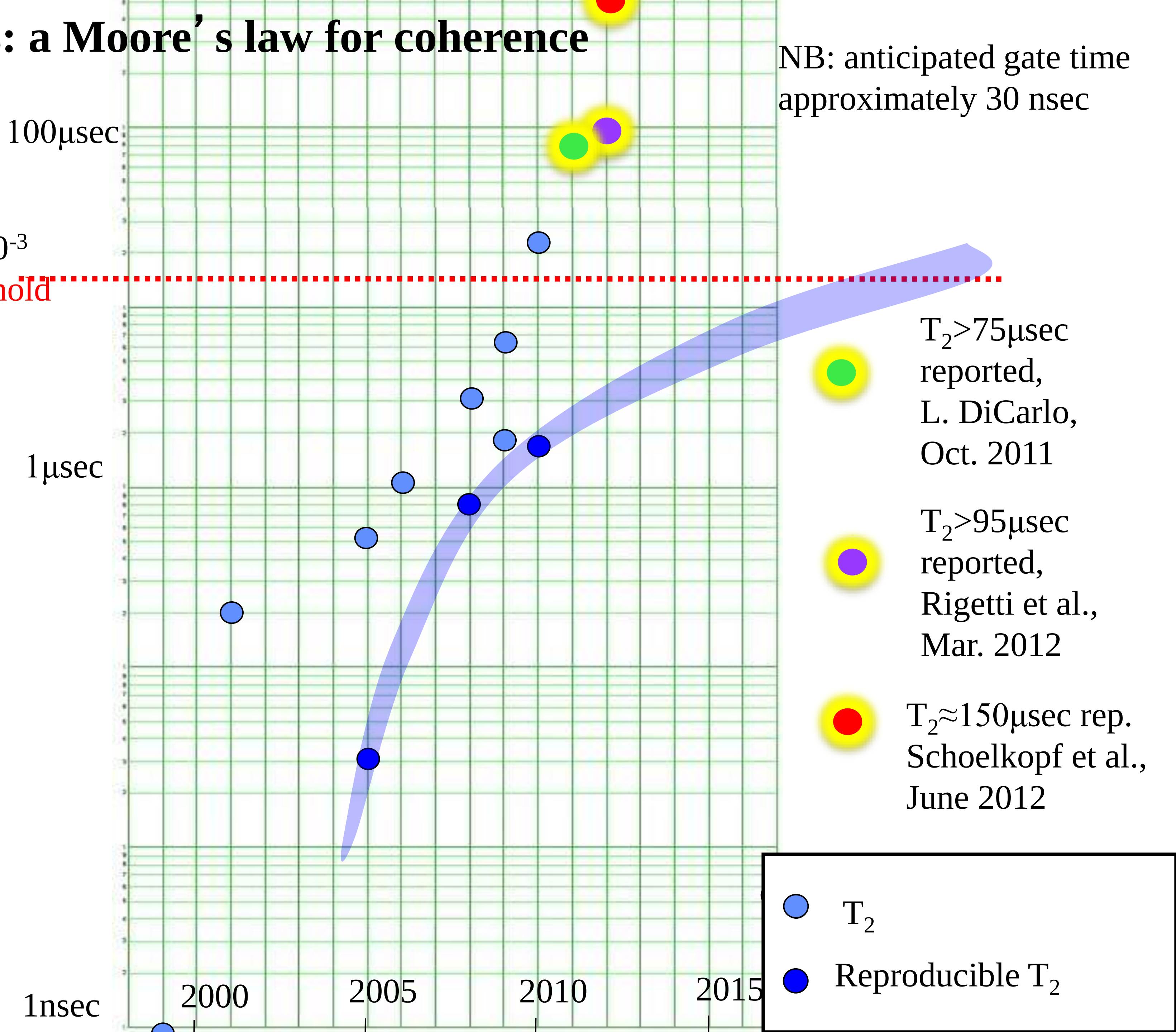
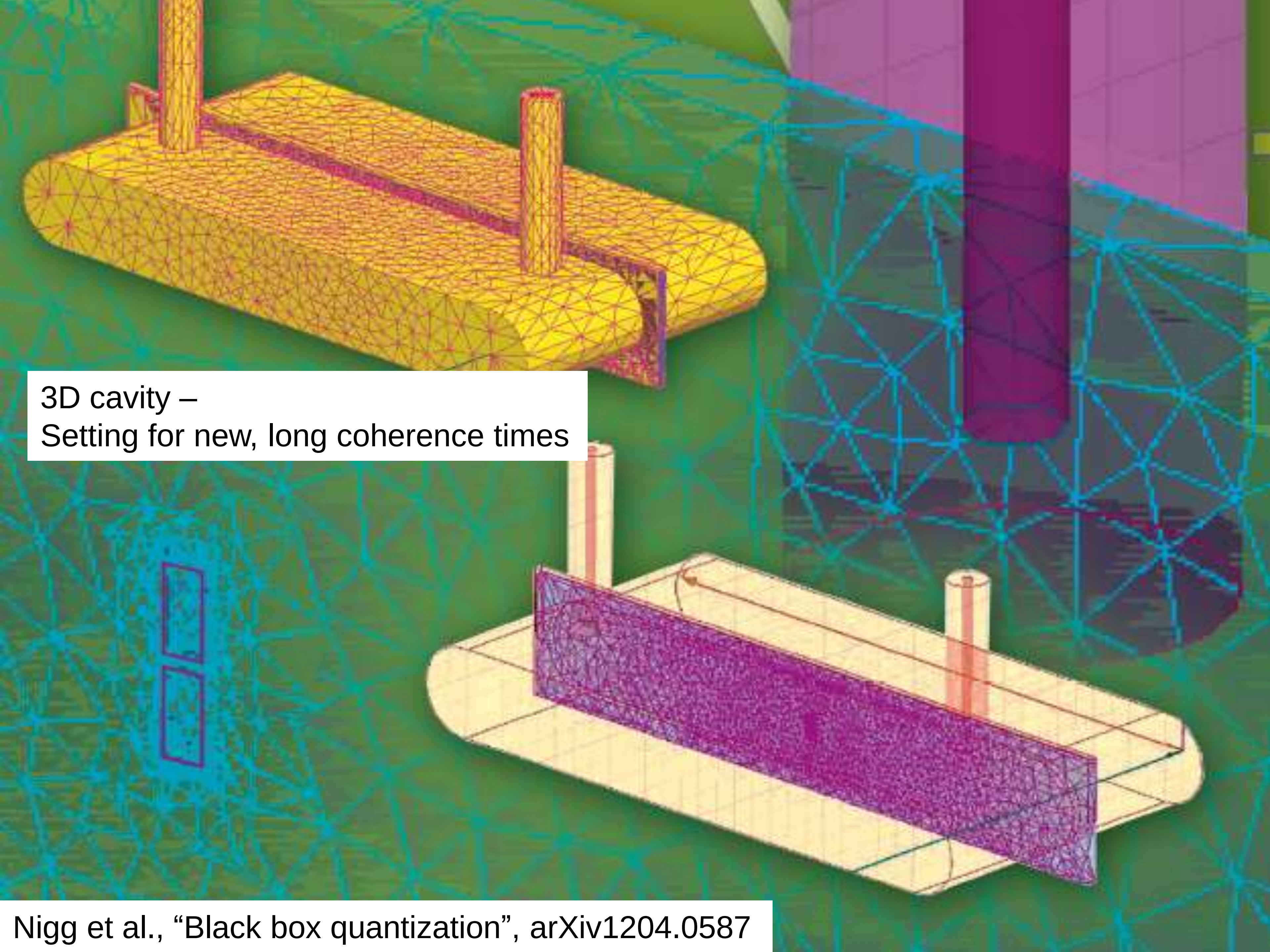




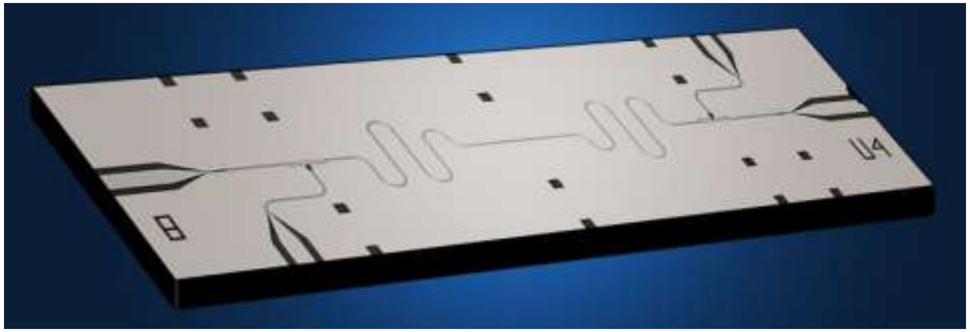
3D cavity –
Setting for new, long coherence times

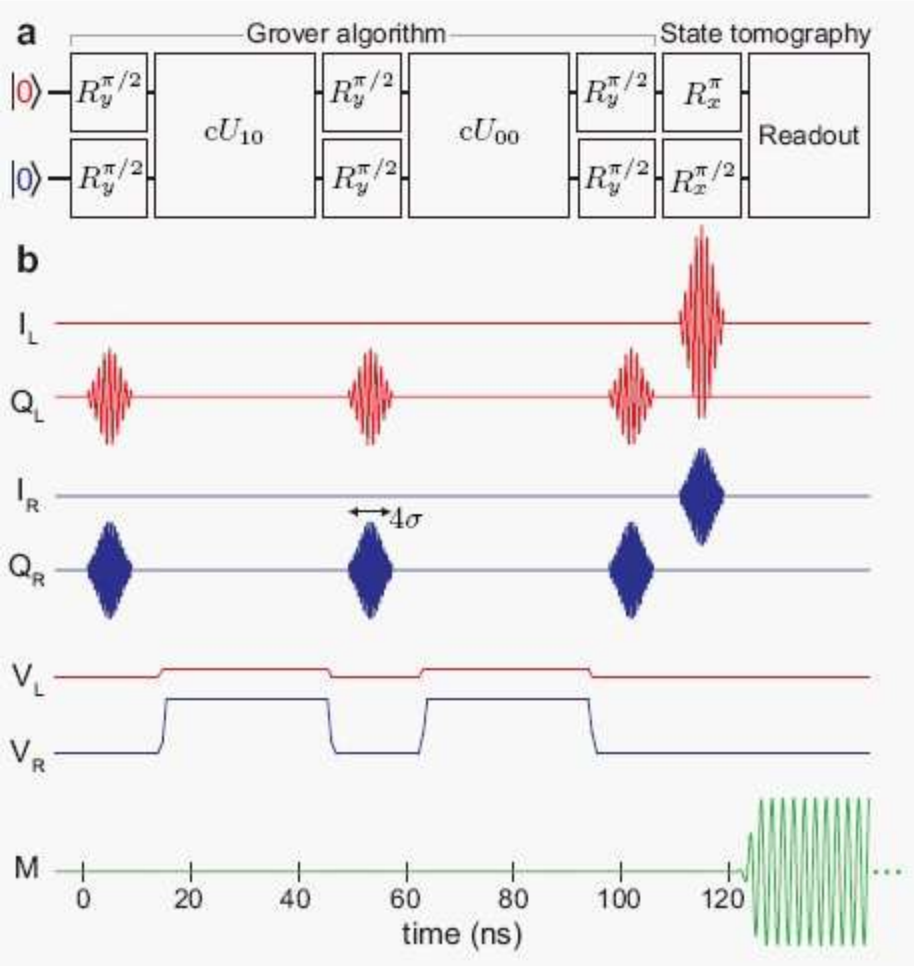
Nigg et al., “Black box quantization”, arXiv1204.0587

Demonstration of Two-Qubit Algorithms with a Superconducting Quantum Processor

L. DiCarlo,[1] J. M. Chow,[1] J. M. Gambetta,[2] Lev S. Bishop,[1] B. R. Johnson,[1] D. I. Schuster,[1] J. Majer,[3] A. Blais,[4] L. Frunzio,[1] S. M. Girvin,[1] and R. J. Schoelkopf[1]
[1] *Departments of Physics and Applied Physics, Yale University, New Haven, CT 06511, USA*

Nature 460, 240-244 (2009)



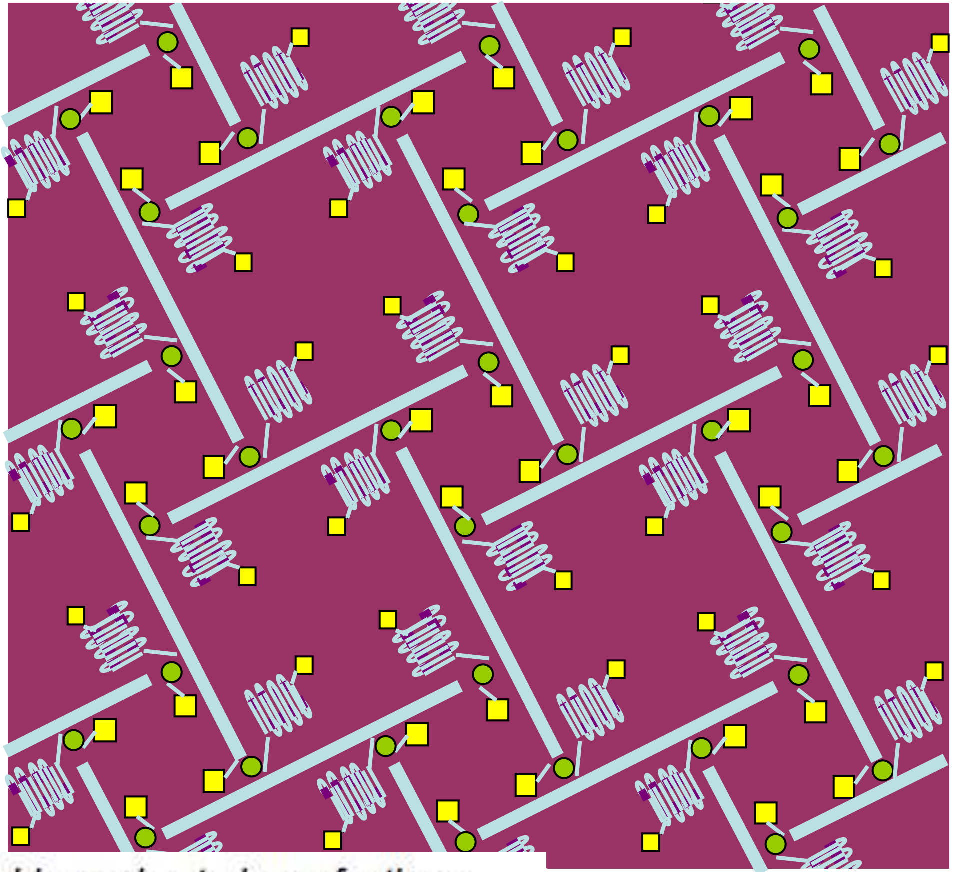


Fidelity above 90% for two qubit gates

Like early NMR experiments, but in scalable system!

Regular square lattice of coupled qubits make an effective architecture for fault tolerance

- Qubits (green) coupled via high-Q superconducting resonators (gray)
- “skew-square” layout of qubits and resonators is one way to achieve abstract square
- Every qubit has a number of controller and sensor lines to be connected to the outside world (gold pads)

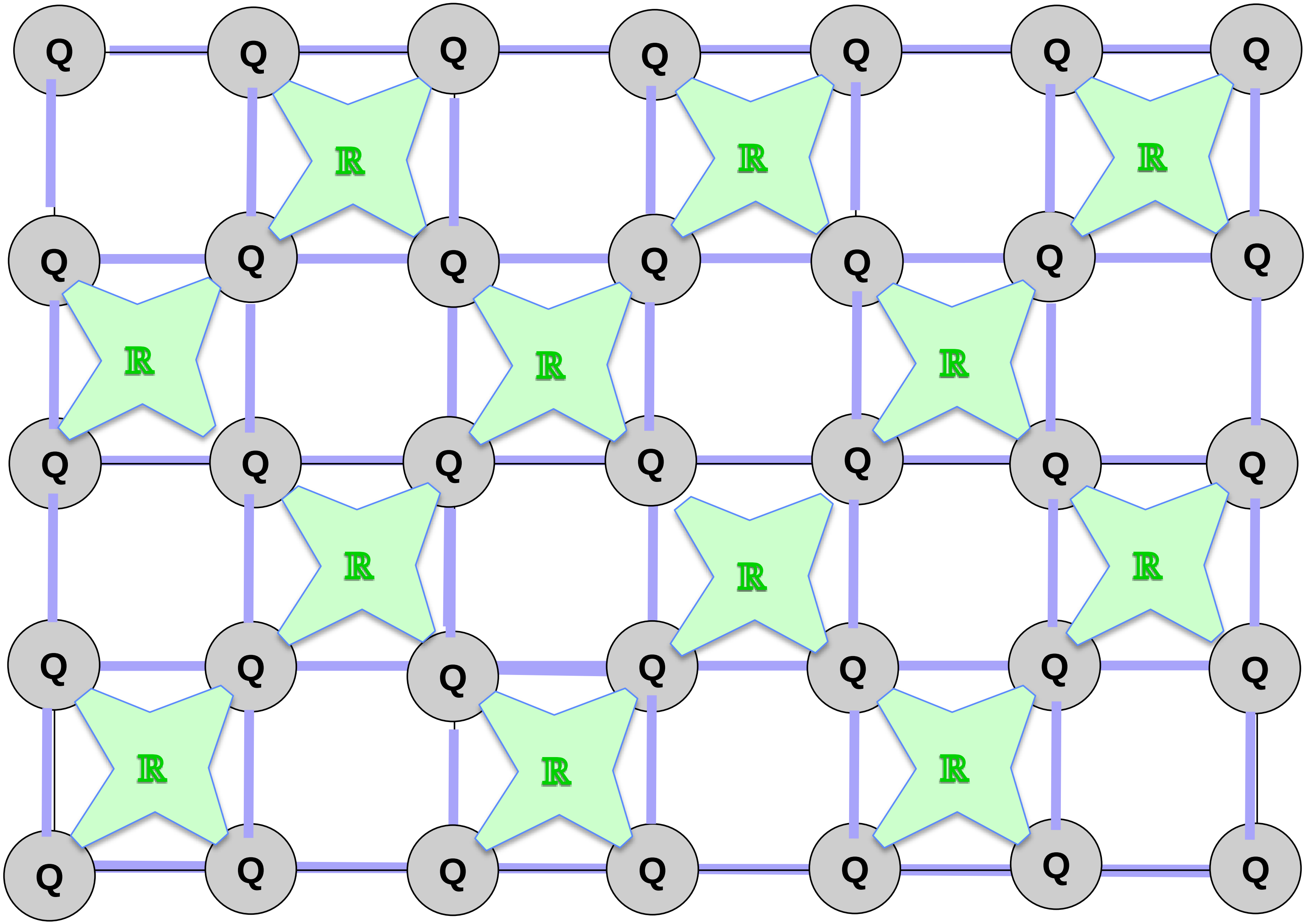


DP. DiVincenzo, “Fault tolerant architectures for superconducting qubits,” Phys. Scr. T **137** (2009) 014020.

“In a machine such as this there are very many other problems due to imperfections. . . . At least some of these problems can be remedied in the usual way by techniques such as error correcting codes . . . But until we find a specific implementation for this computer, I do not know how to proceed to analyze these effects.”

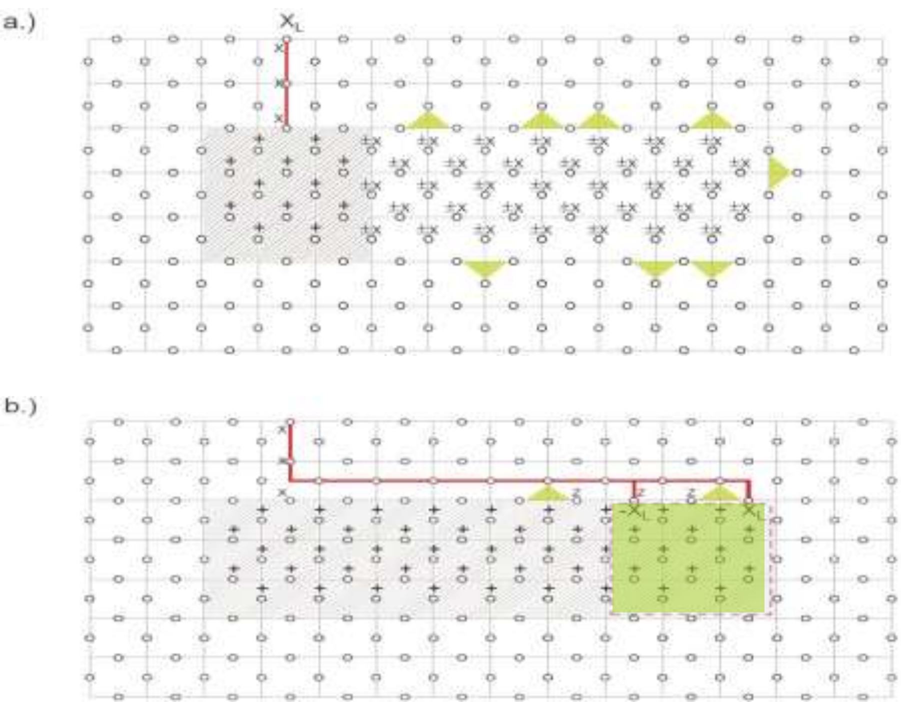
R.P. Feynman
“Quantum Mechanical Computers”
Optics News, February 1985

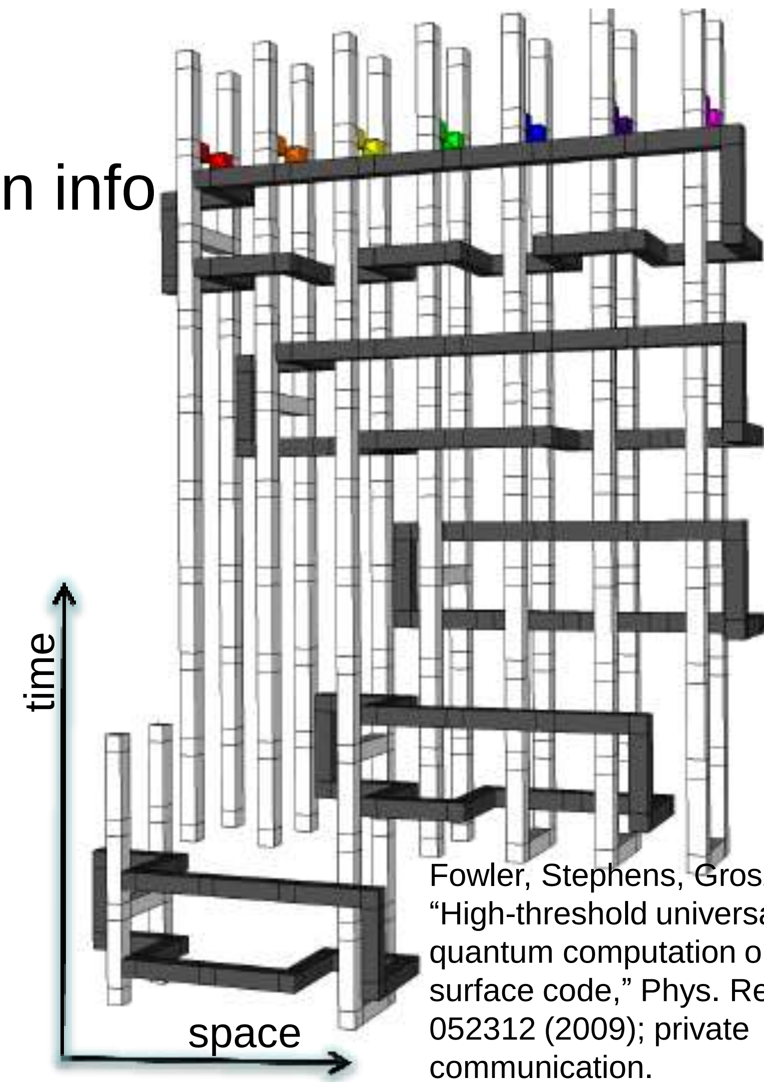
Surface code error correction: skew-square cavity arrangement is topologically equivalent to this:



Fault tolerant algorithms with surface code

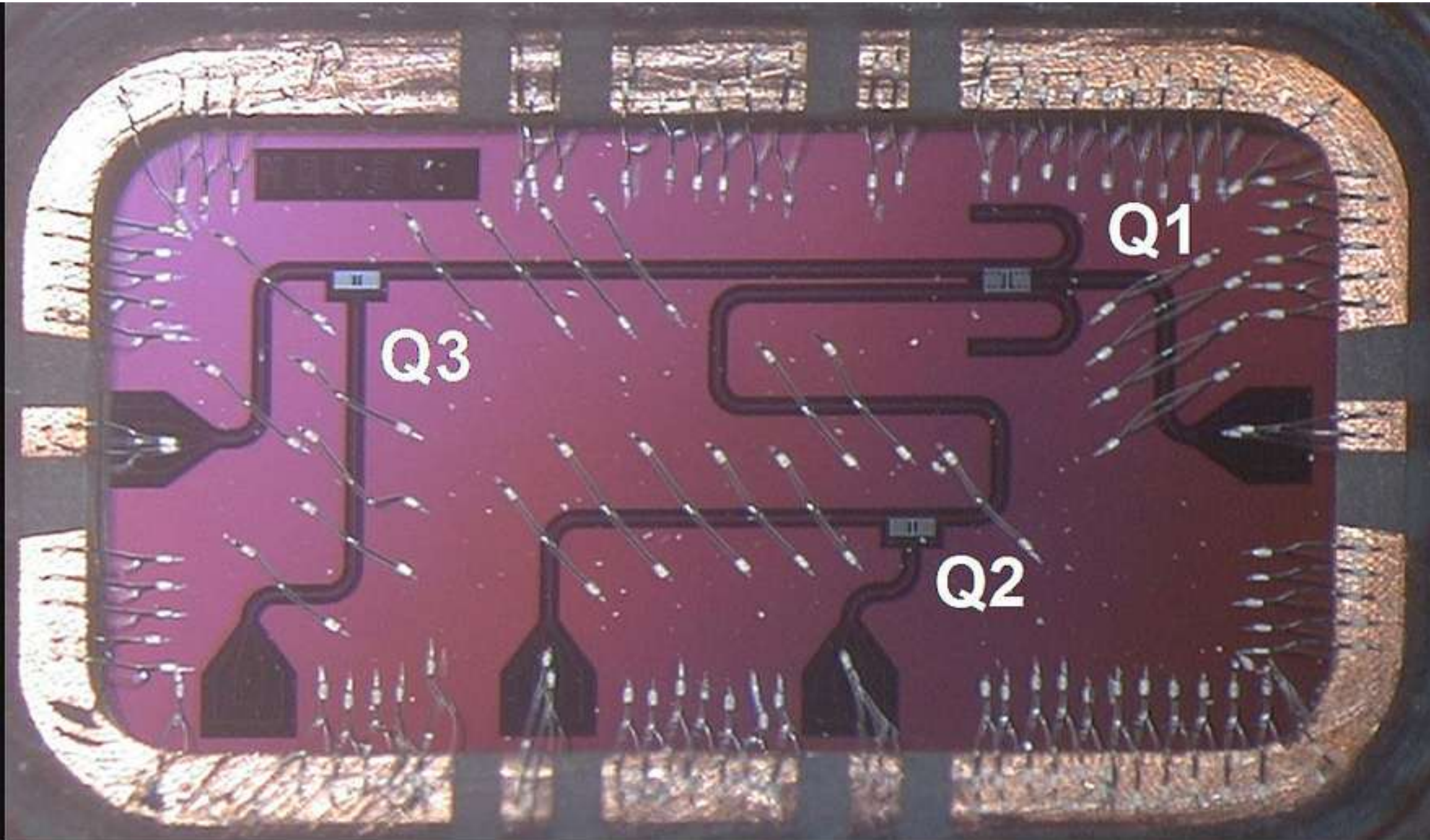
- Do repetitive pattern in large patches, except for holes where nothing is done
- Holes define qubits; algorithm performed by braiding holes
- measurements give error correction info
- theoretical threshold is 0.75%





Fowler, Stephens, Groszkowski, “High-threshold universal quantum computation on the surface code,” Phys. Rev. A 80, 052312 (2009); private communication.

IBM (Feb. 2012) – 3 qubit structure, start of scalability?



2D Superconducting qubit

By IBM Research ★ Favorite 💬 Comment

IBM (Feb. 2013) – a year of hard work

B 4 Control of Superconducting Qubits [1]

Jay M. Gambetta

IBM T.J. Watson Research Center

Yorktown Heights, New York 10598, USA

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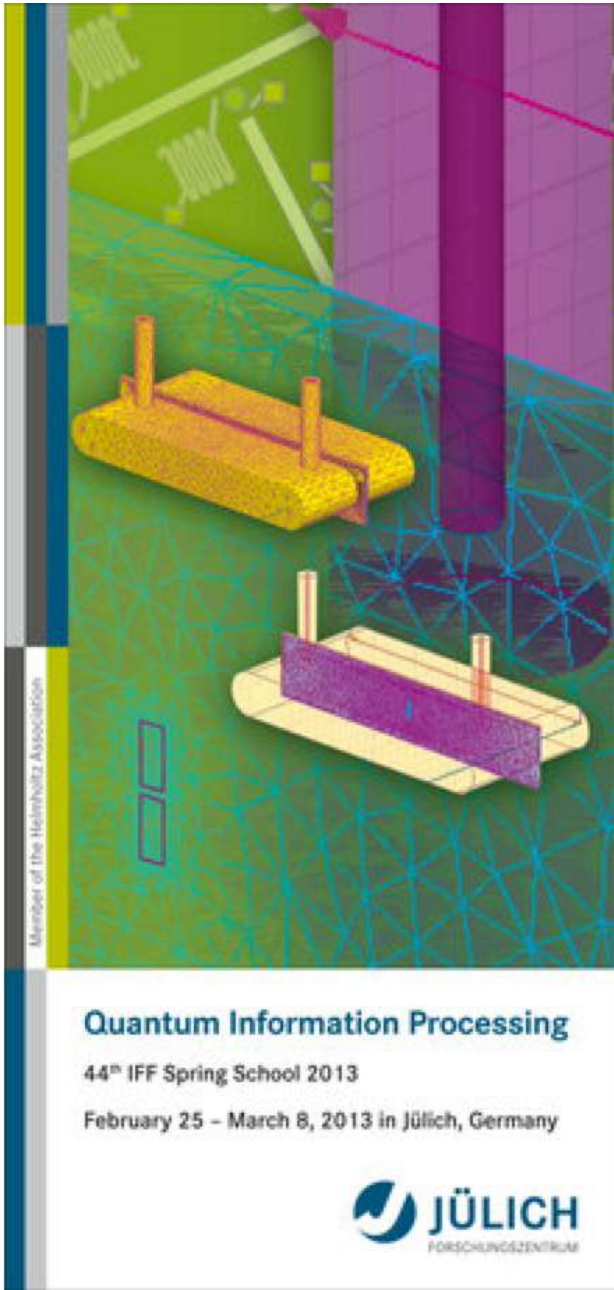

Gambetta summary of IBM results

Gate	Reference	F_s (Bell)	Process tomography F_g	Benchmarking F_g
iSwap	[35]	0.91	0.92	-
iSwap (phase)	[49]	-	0.70	-
cPhase	[36, 29]	0.94	-	-
cPhase (phase)	[50]	-	0.76	-
ZX_{90}	[5]	0.983	0.968 (raw) 0.95 (mle)	-
ZX_{90} echo	[43]	-	0.883 (mle) 0.880 (mle)	0.935
bSwap	[44]	> 0.99	0.873 (raw) and 0.80 (mle)	-
sideband	[48]	0.58	-	-

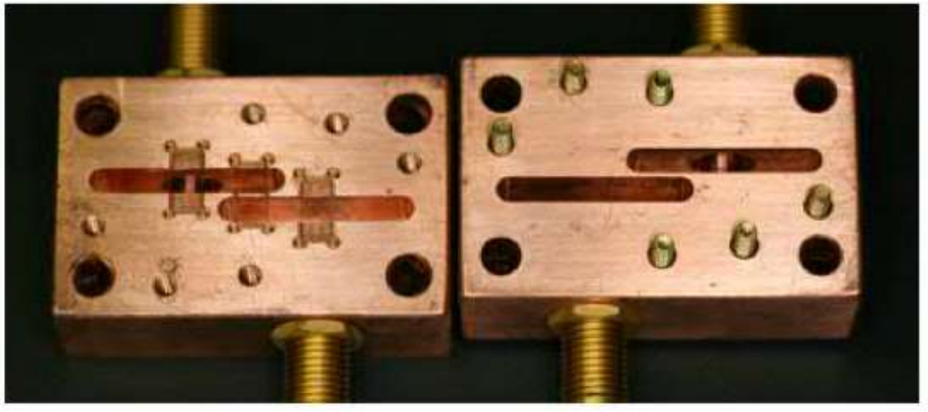

Fig. 8: *A copper enclosure consisting of two cavities (and three transmons). The transmons are fabricated on individual sapphire or silicon chips and are independently pre-characterization and chosen for there desired application.*

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- A short history of quantum effects in electric circuits
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R.P. Feynman
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Masters/PhD/postdocs available!
http://www.physik.rwth-aachen.de/institute/institut-fuer-quanteninformation/
(G: IQI Aachen)

Prospects for Superconducting Qubits

David DiVincenzo

24.01.2013

QIP Beijing

Prospects for Superconducting Qubits

Outline

- Superconducting qubits: a quantum Moore's law
- Error correction – using massive redundancy
- Using the surface code
- The architecture of a large quantum computer – road map
- Going "off road":
  - New error correction ideas
  - Direct multiqubit parity measurements

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R.P. Feynman
"Quantum Mechanical Computers"
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A development of 1996-7:

Quantum error correction with imperfect gates

A. Yu. Kitaev

L.D.Landau Institute for Theoretical Physics,
117940, Kosygina St. 2
e-mail: kitaev@itp.ac.ru

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Abstract

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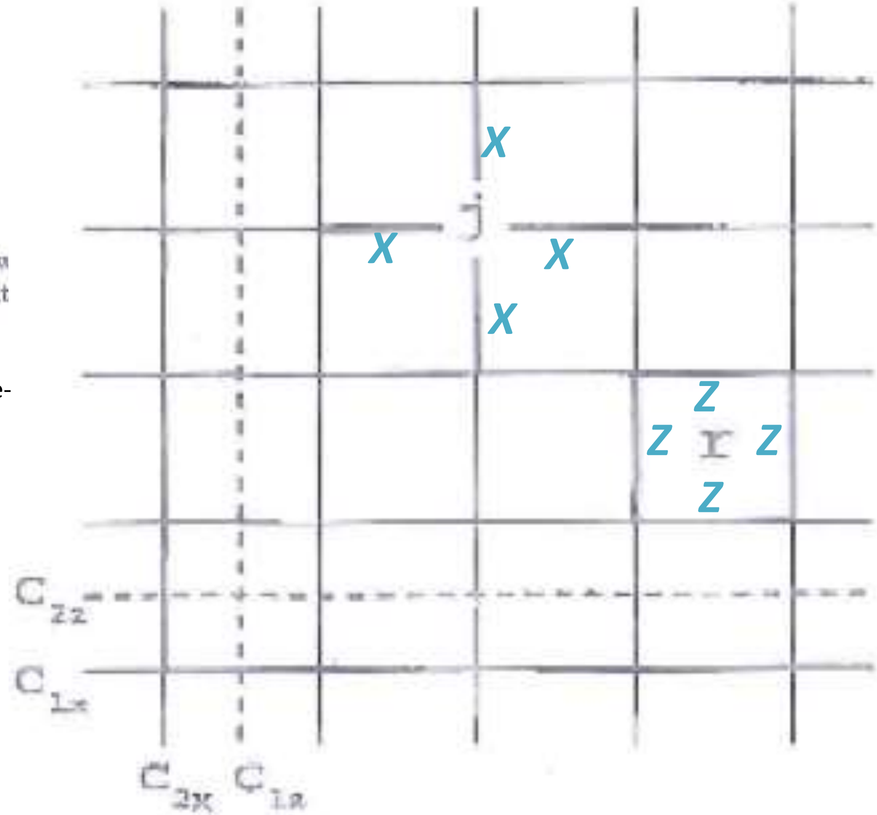


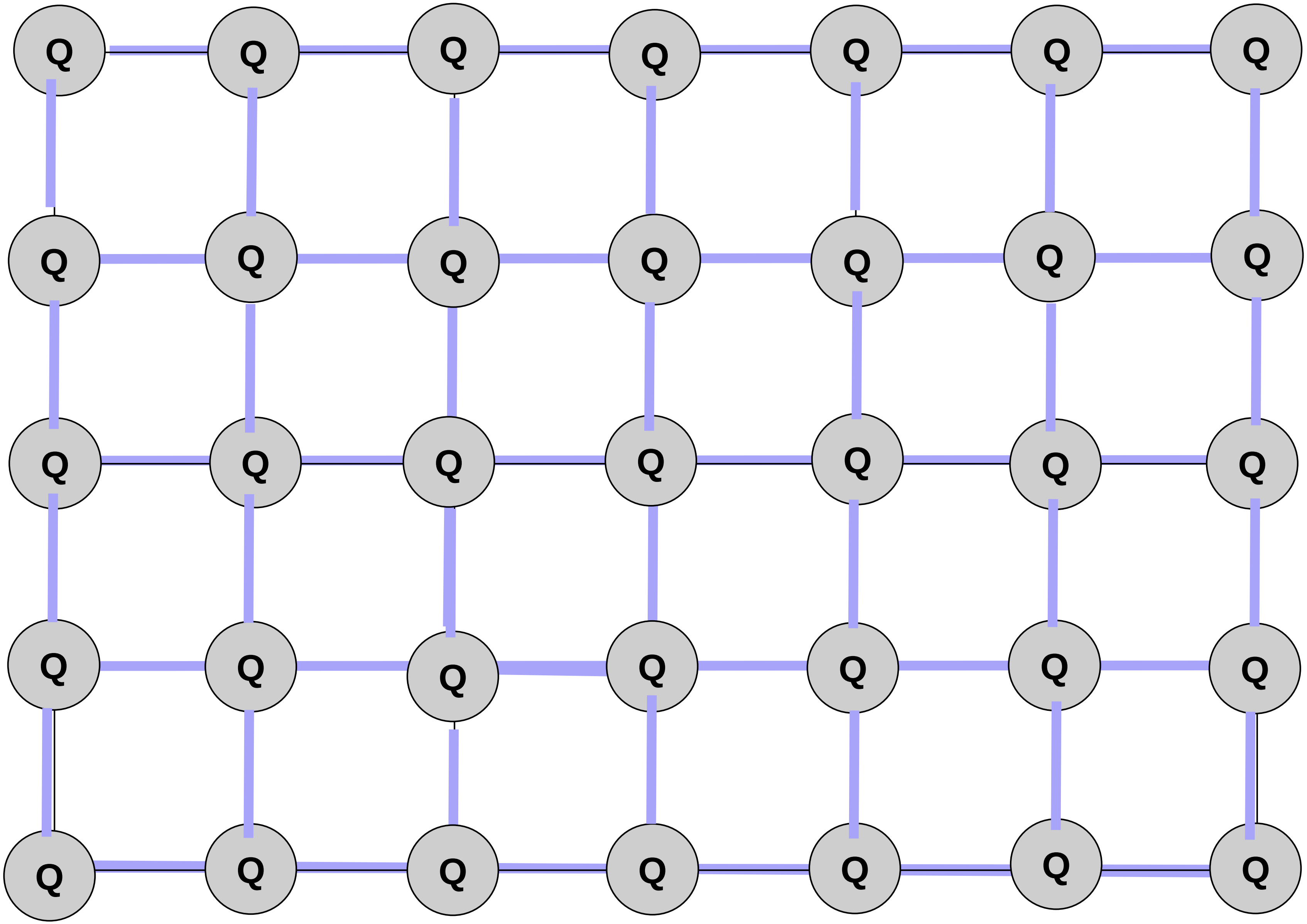
Figure 1: The toric code TOR(5).

Stabilizer generators XXXX, ZZZZ;

Stars and plaquettes of interesting 2D lattice Hamiltonian model

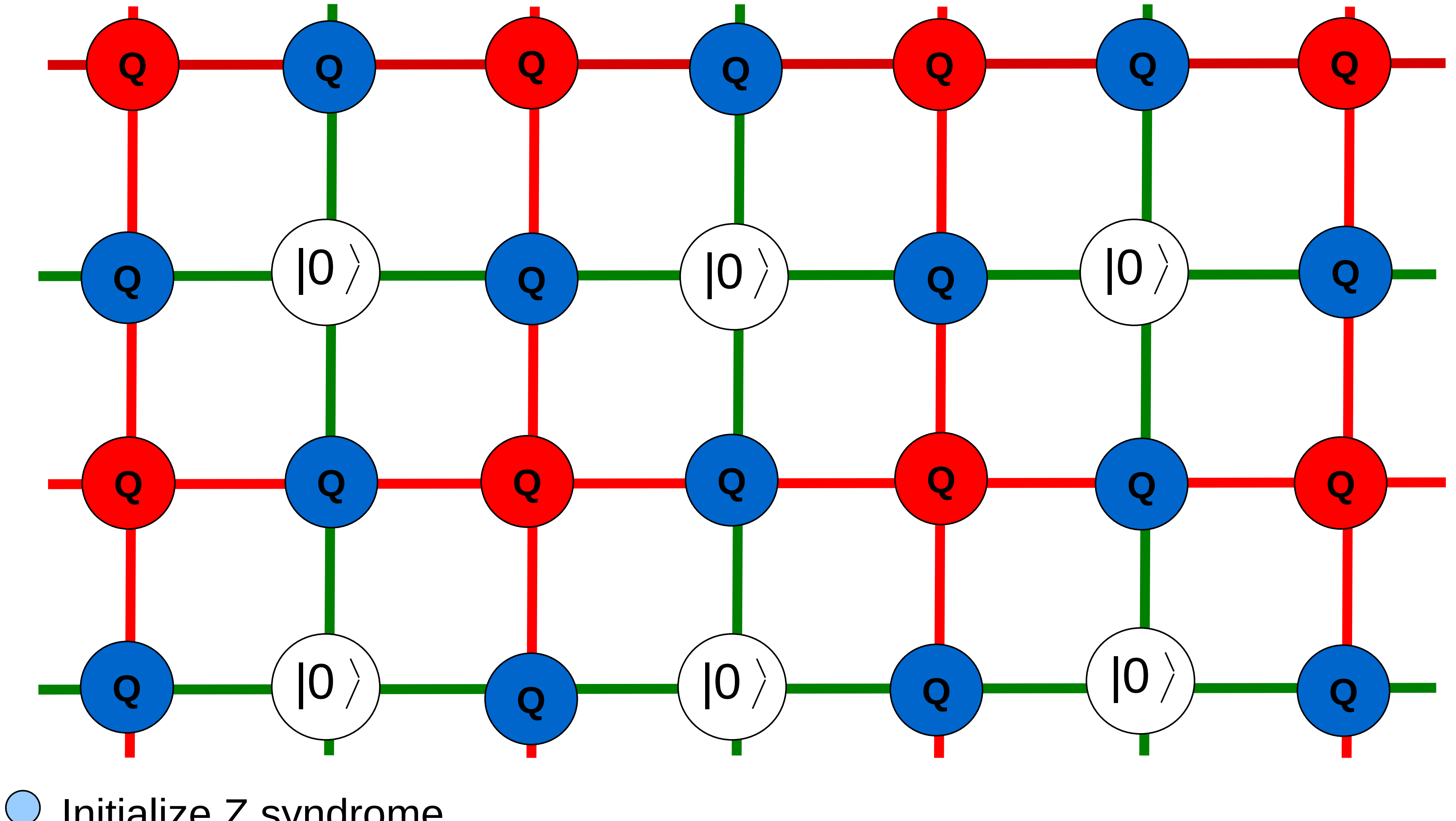
Toric Code/Surface Code

Surface code error correction: qubits (abstract) in fixed 2D square arrangement (“sea of qubits”), only nearest-neighbor coupling are possible



Colorized thanks to
Jay Gambetta and John Smolin

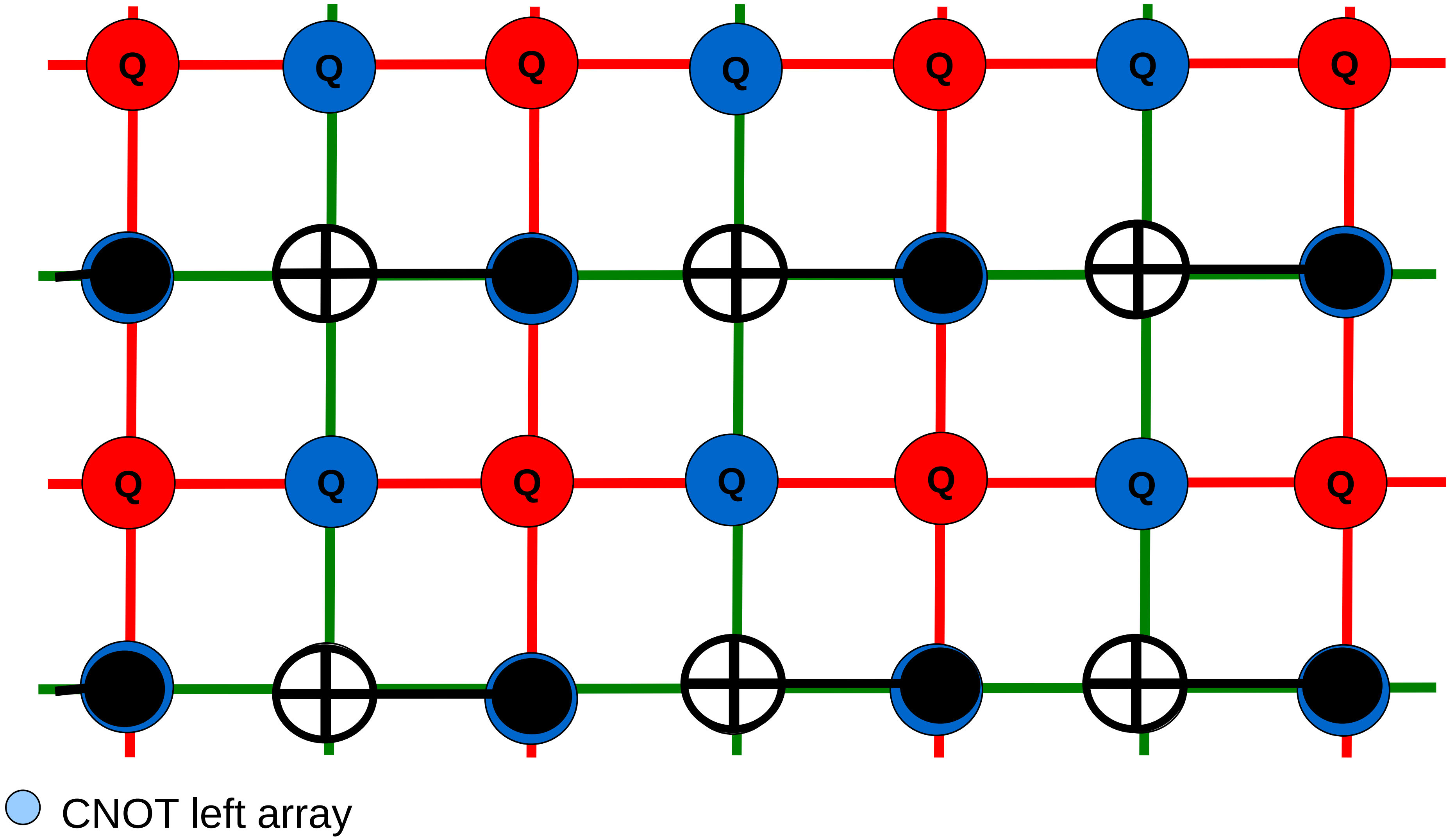
Surface code



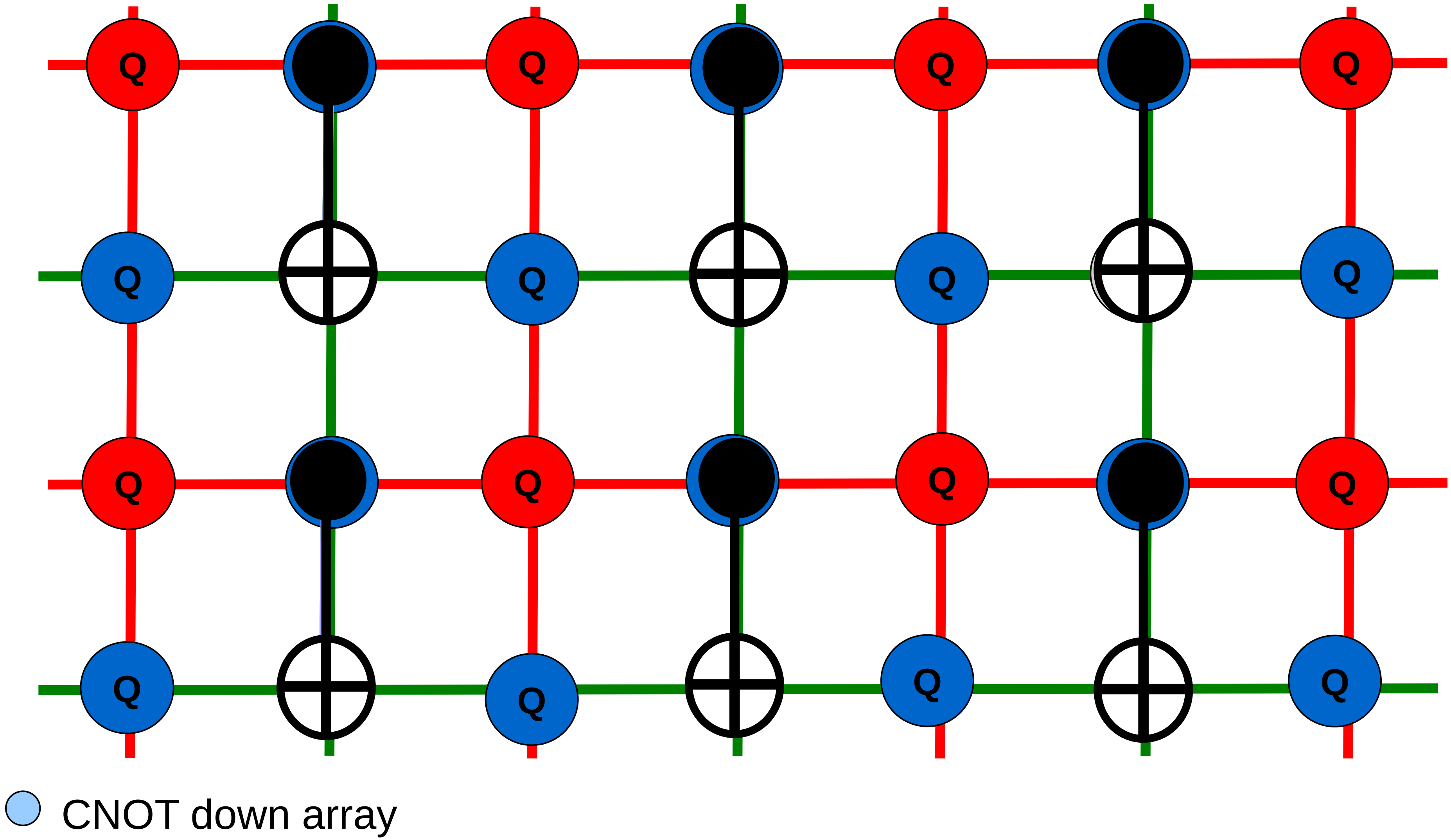
Initialize Z syndrome qubits to $|0\rangle$

Implementing the "surface code":
-- in any given patch, independent of the quantum algorithm to be done:

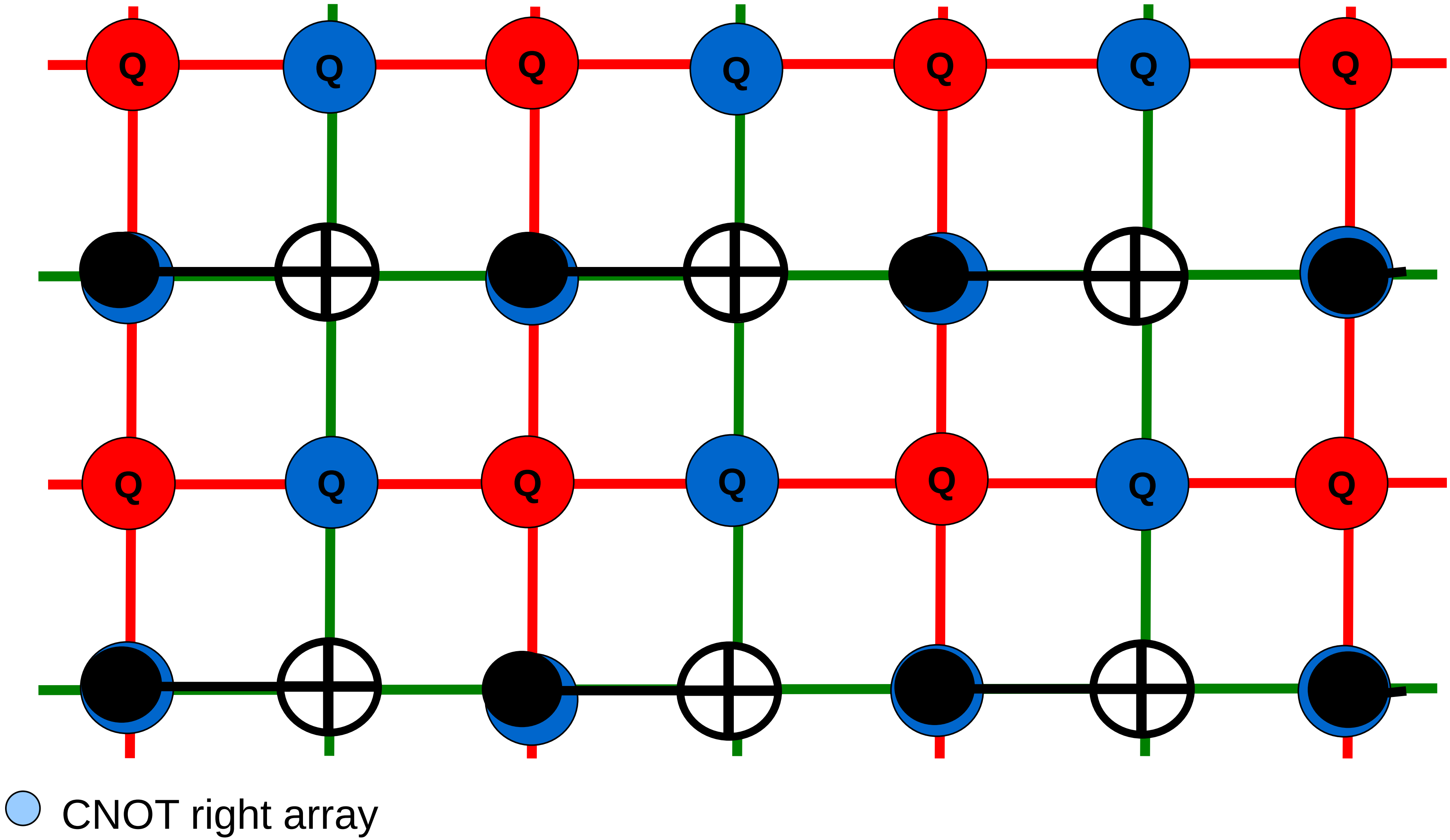
Surface code



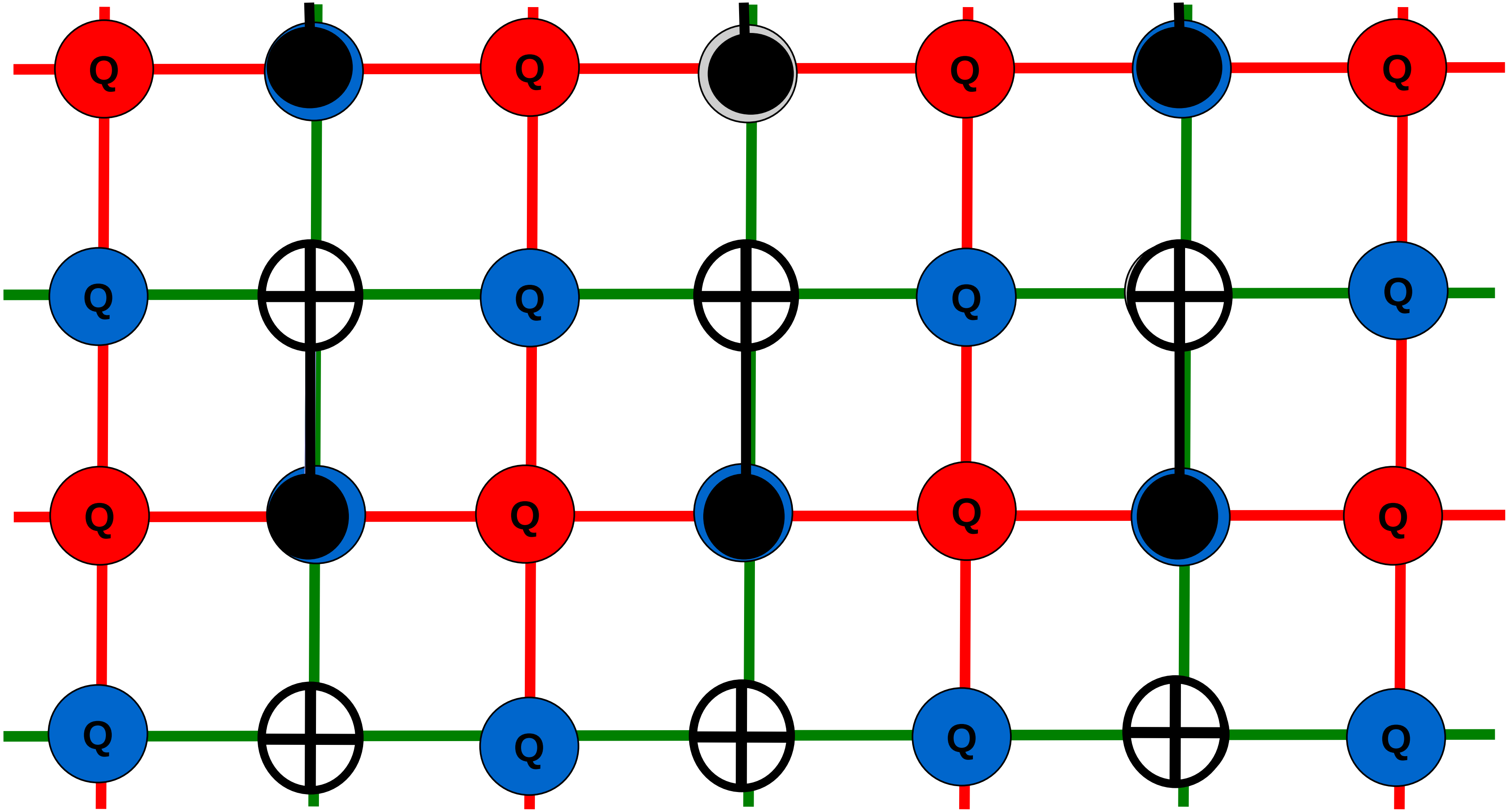
Surface code



Surface code

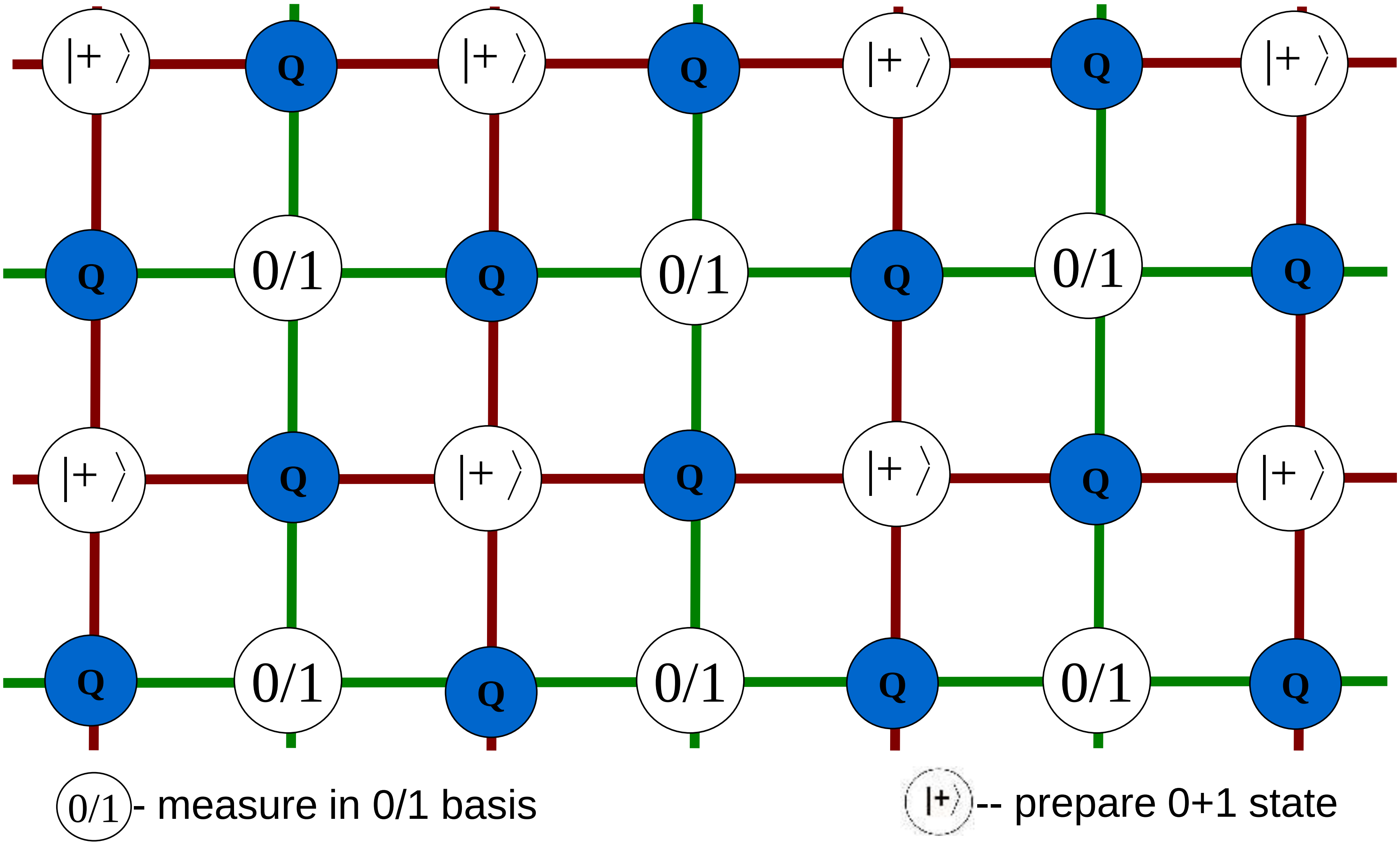


Surface code fabric



CNOT up array

Surface code fabric



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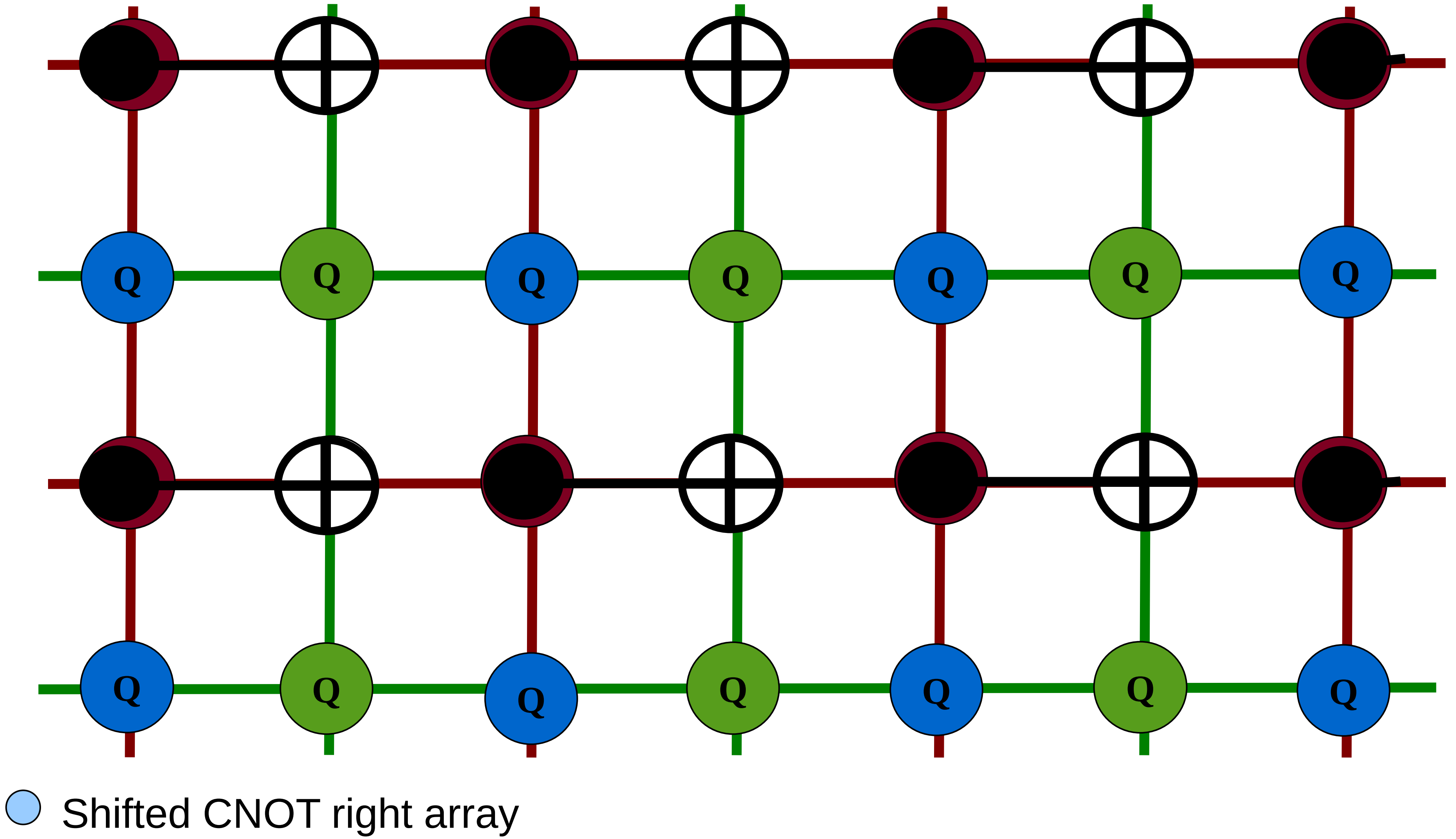
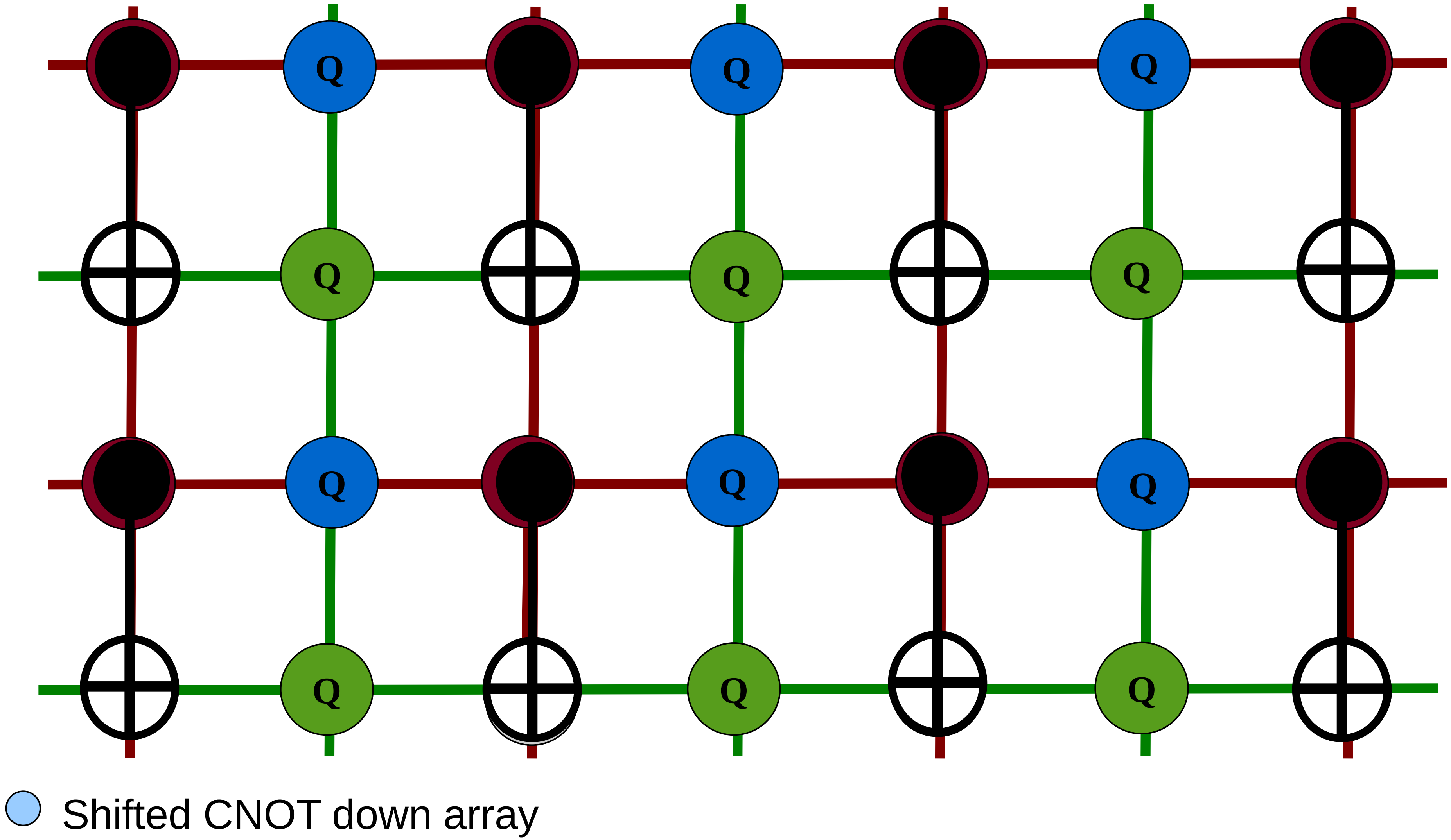
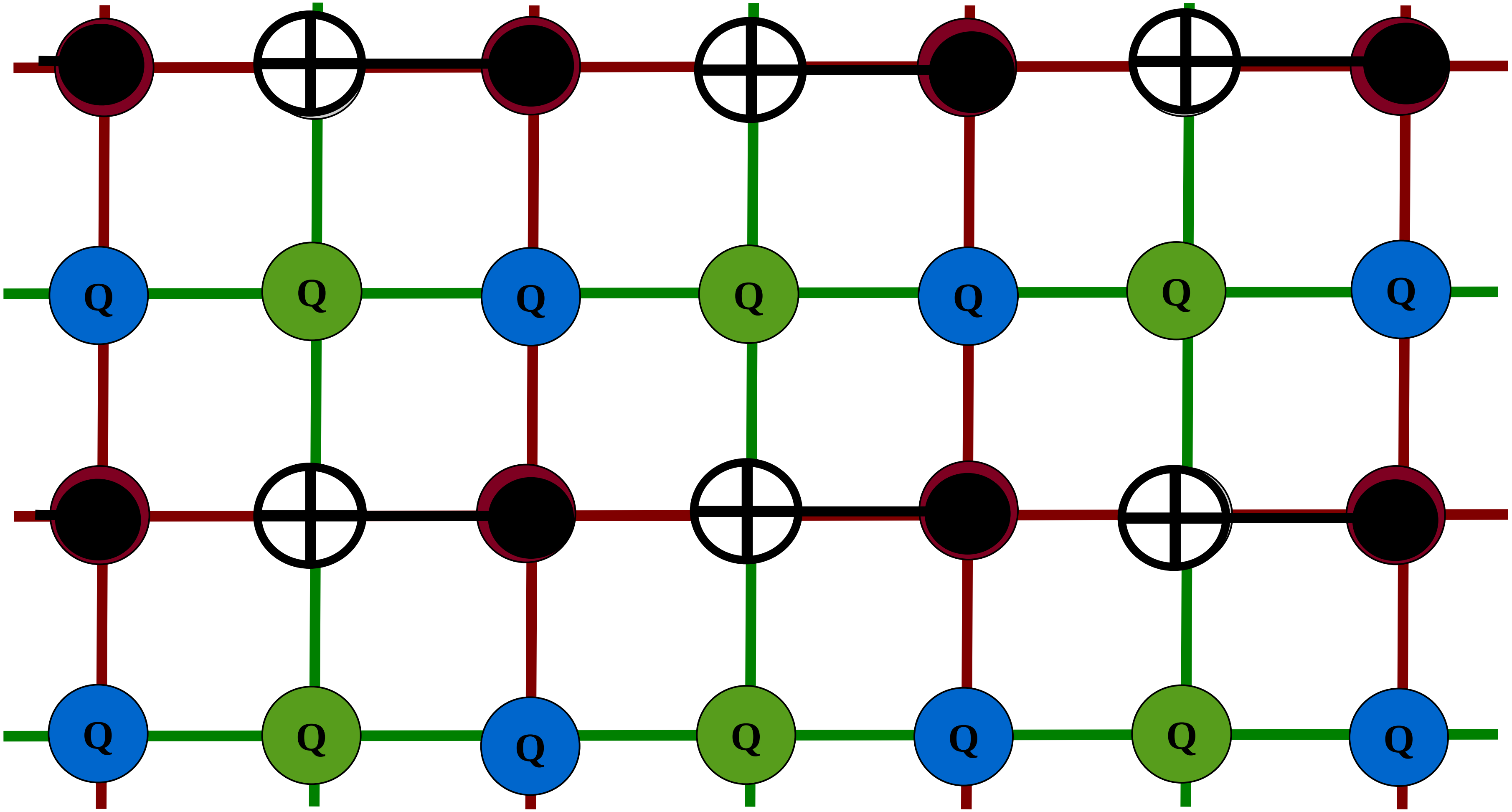


Surface code



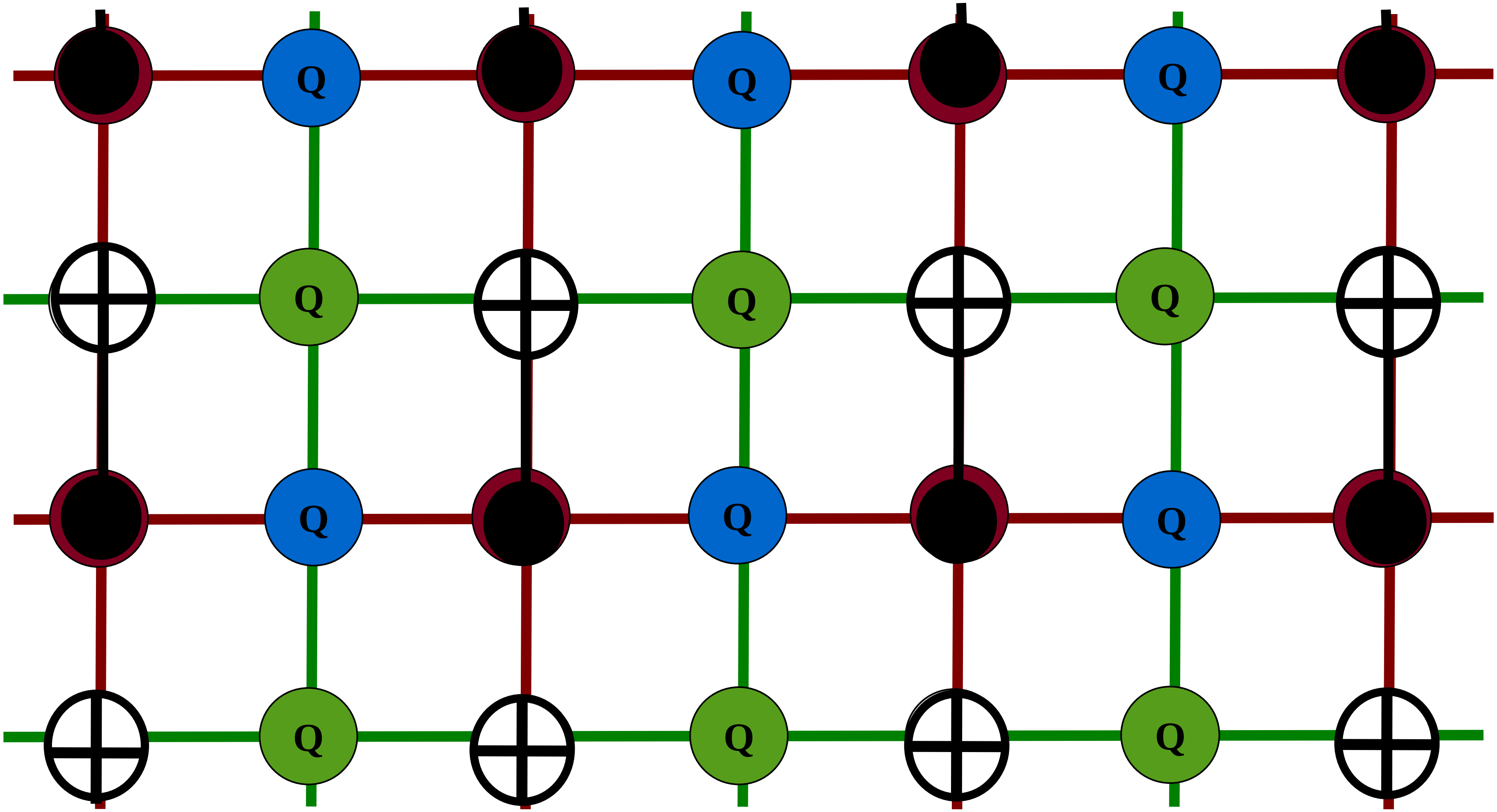
Shifted CNOT down array

Surface code



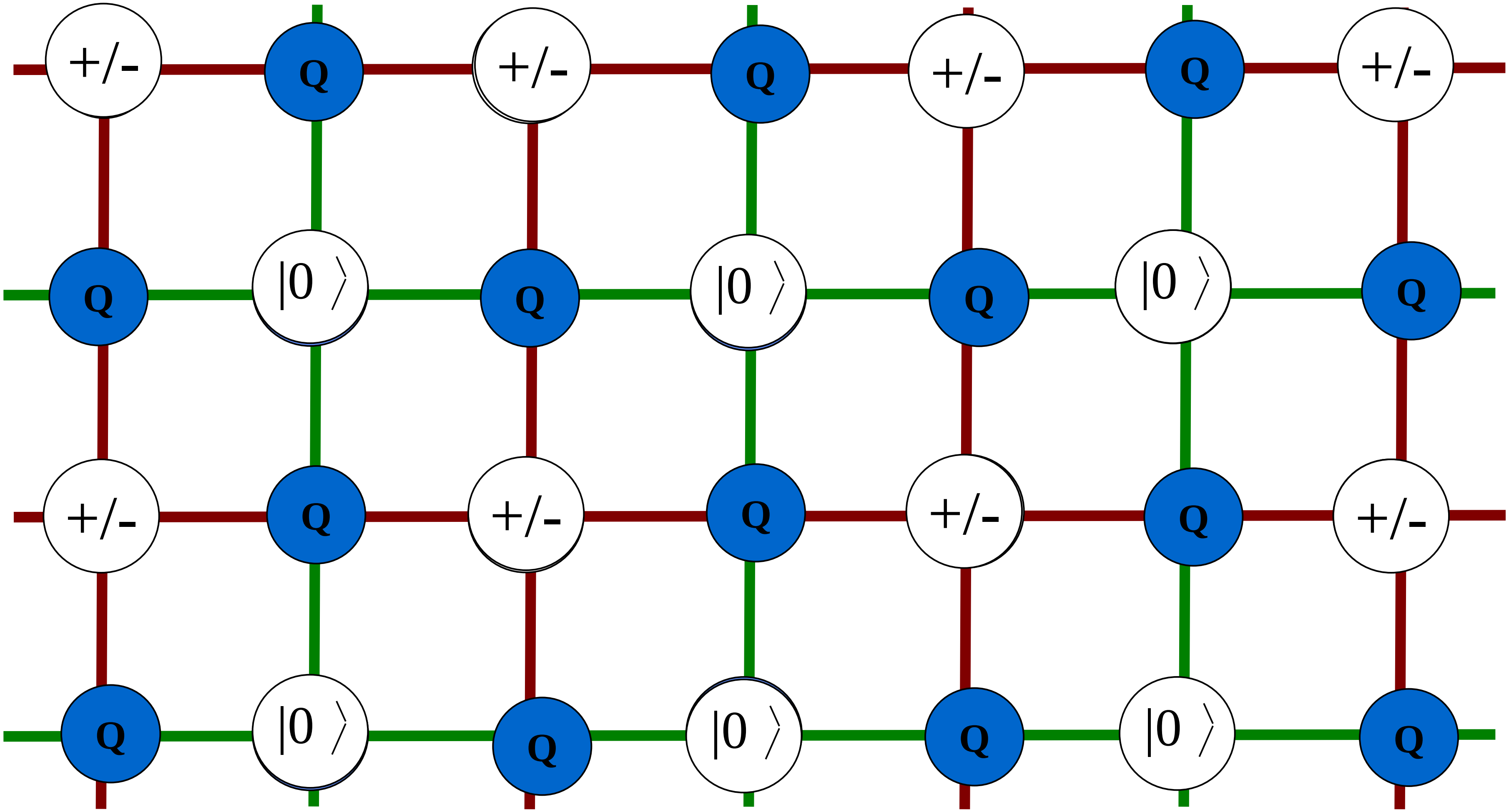
Shifted CNOT left array

Surface code



Shifted CNOT up array

Surface code fabric



- Repeat over and over....

Regular square lattice of coupled qubits make an effective architecture for fault tolerance

- Qubits (green) coupled via high-Q superconducting resonators (gray)
- "skew-square" layout of qubits and resonators is one way to achieve abstract square
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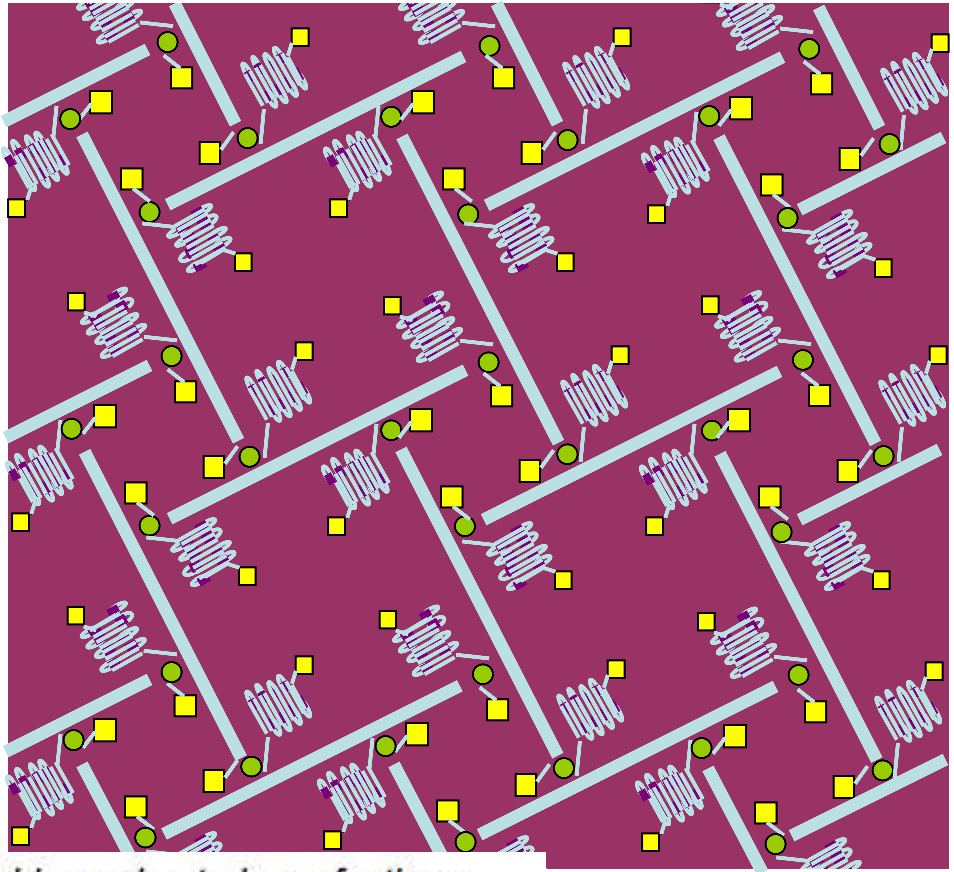

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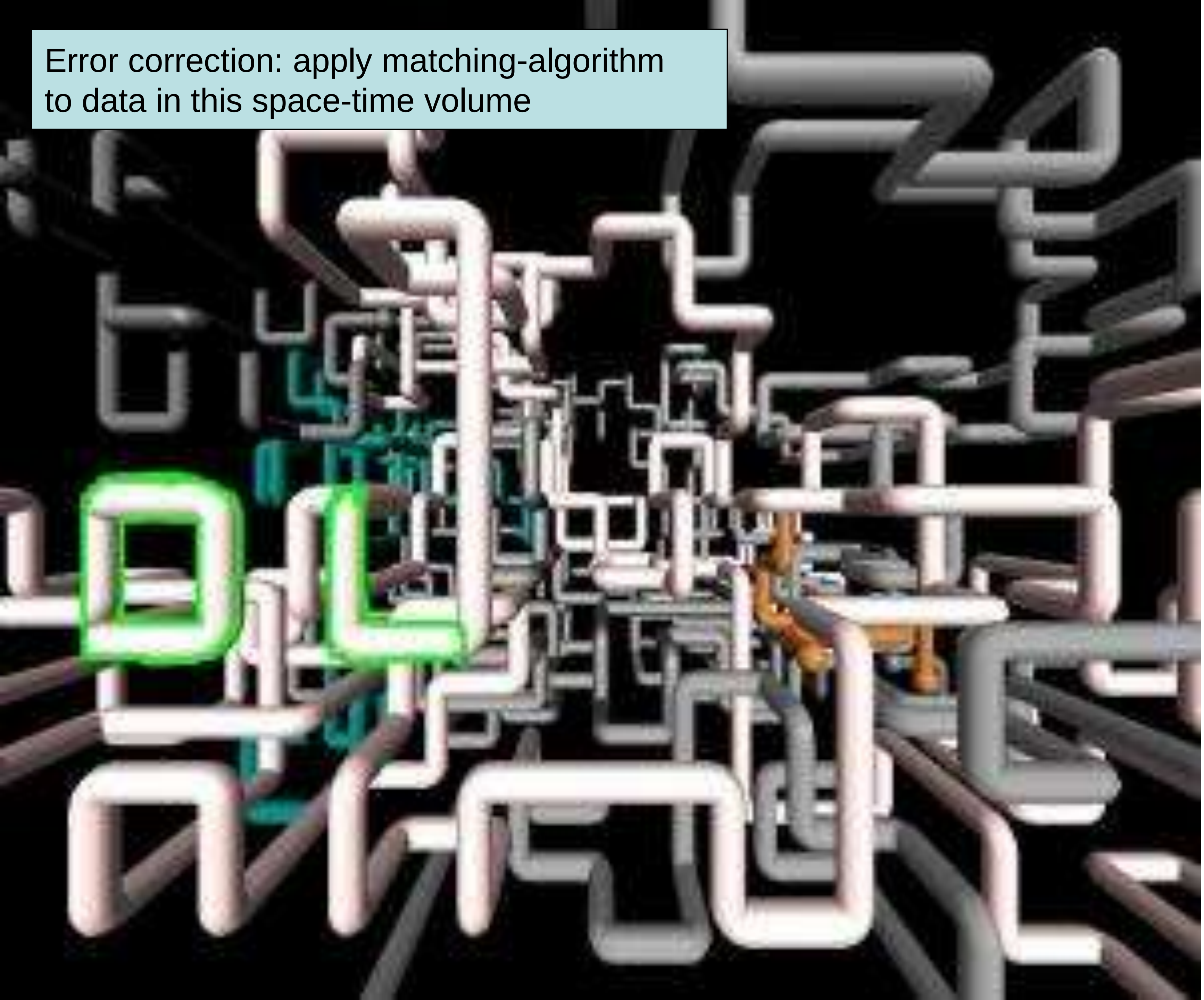
Optics News, February 1985

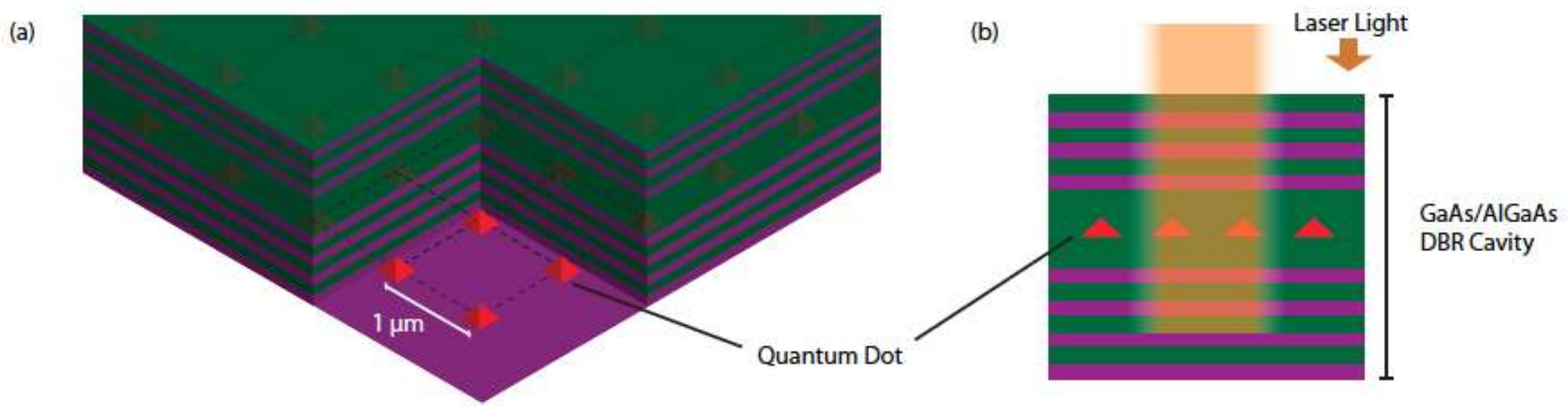
DP. DiVincenzo, "Fault tolerant architectures for superconducting qubits," Phys. Scr. T **137** (2009) 014020.

Error correction: apply matching-algorithm to data in this space-time volume



A different notional architecture: optically controlled quantum dots (NOT my preferred embodiment!)

NC Jones et al., "Layered Architecture For Quantum Computing," Phys Rev X **2**, 031007 (2012)



Some interesting numbers

Computing Resource		Shor's Algorithm (1024-bit)	Molecular Simulation (alanine)
Layer 5	Application qubits	6144	6650
	Circuit depth (Toffoli)	1.68×10^{8}	1.27×10^{9}
Layer 4	Log. distillation qubits	66564	15860
	Logical clock cycles	5.21×10^{9}	3.94×10^{10}
Layer 3	Code distance	31	31
	Error per lattice cycle	2.58×10^{-20}	2.58×10^{-20}
Layer 2	Virtual qubits	4.54×10^{8}	1.40×10^{8}
	Error per virtual gate	1.00×10^{-3}	1.00×10^{-3}
Layer 1	Quantum dots (area on chip)	4.54×10^{8} (4.54 cm^2)	1.40×10^{8} (1.40 cm^2)
Execution time (est.)		**1.81 days**	**13.7 days**

Prospects for Superconducting Qubits

Outline

- Superconducting qubits: a quantum Moore's law
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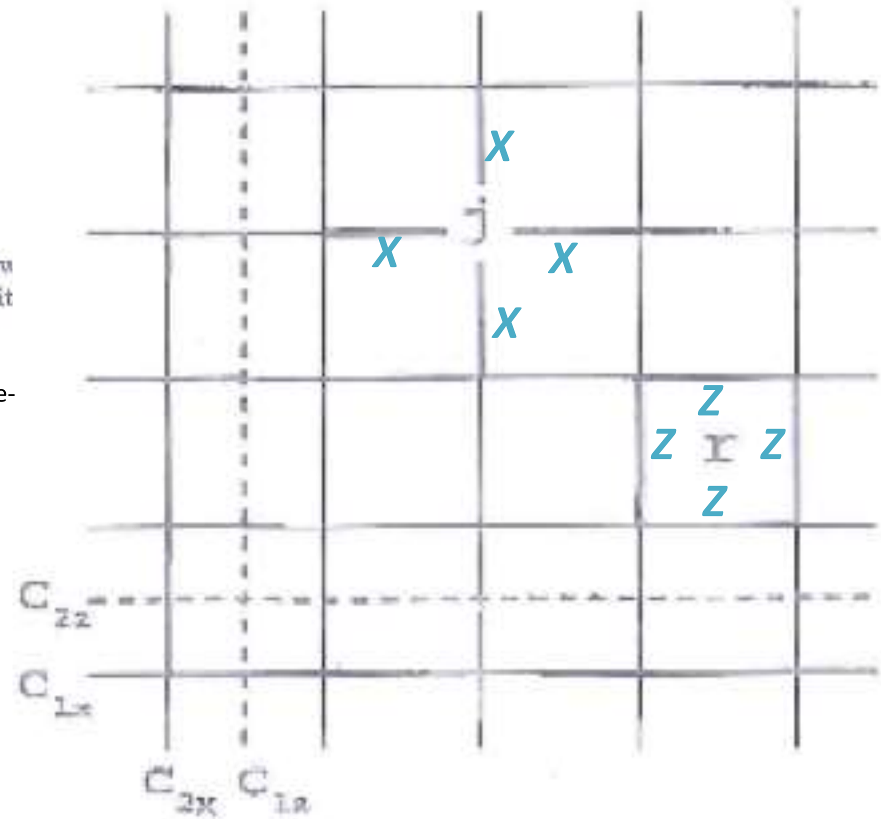

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Subsystem surface codes with three-qubit check operators

1207.1443

Sergey Bravyi,[1] Guillaume Duclos-Cianci,[2] David Poulin,[2] and Martin Suchara[3]

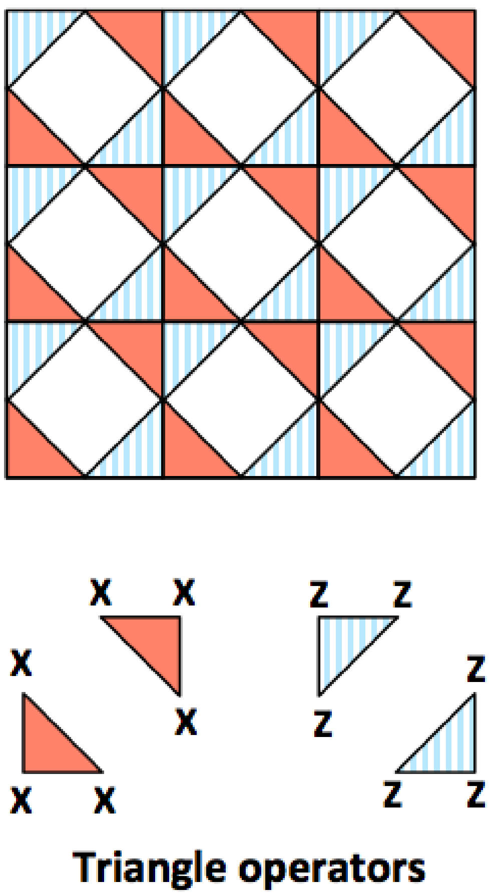


FIG. 1: Subsystem toric code. Qubits live at vertices and centers of edges of the regular square lattice.

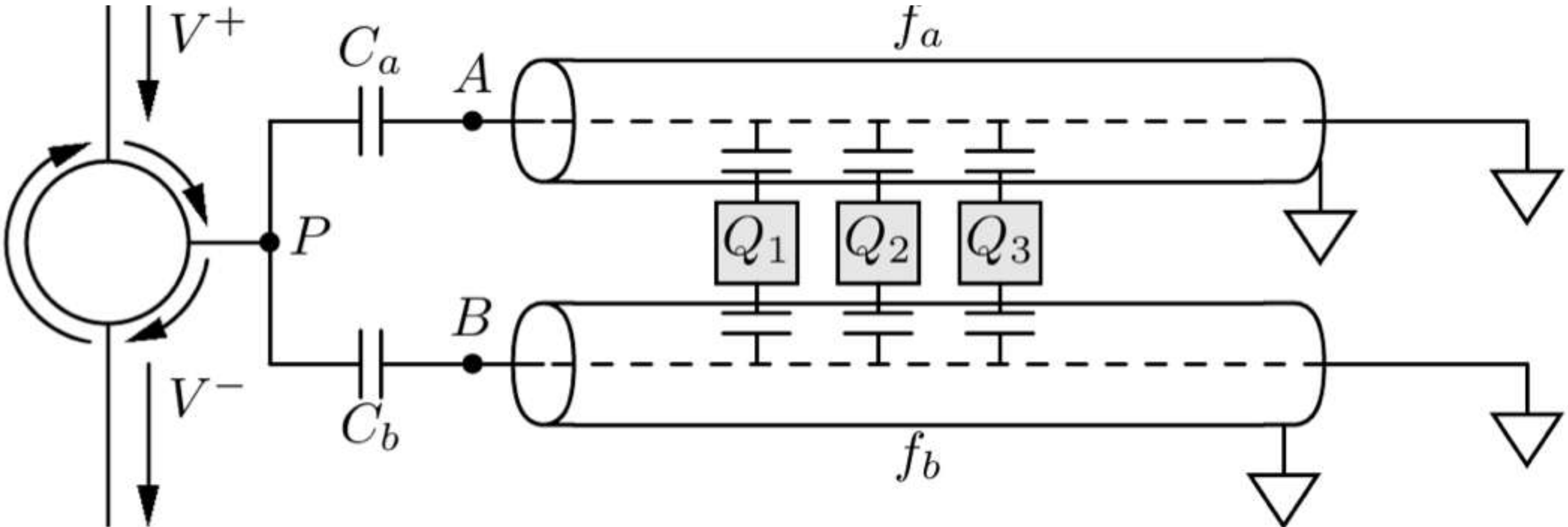
Messages:

- ZZZ rather than ZZZZ
- threshold almost unchanged (1%)
- ZZZ specifies a joint *measurement*, not necessarily a *circuit* – can be done directly?

arXiv:1205.1910

Multi-qubit parity measurement in circuit quantum electrodynamics

David P. DiVincenzo and Firat Solgun



- No gate action among the three qubits
- Three qubits coupled dispersively to each of two nearly degenerate resonant modes
- Measurement by reflectometry: tone in at + port, detect phase of tone out at – port
- Designed as quantum eraser: measures only ZZZ (parity)

arg(r(ω)) for different qubit states

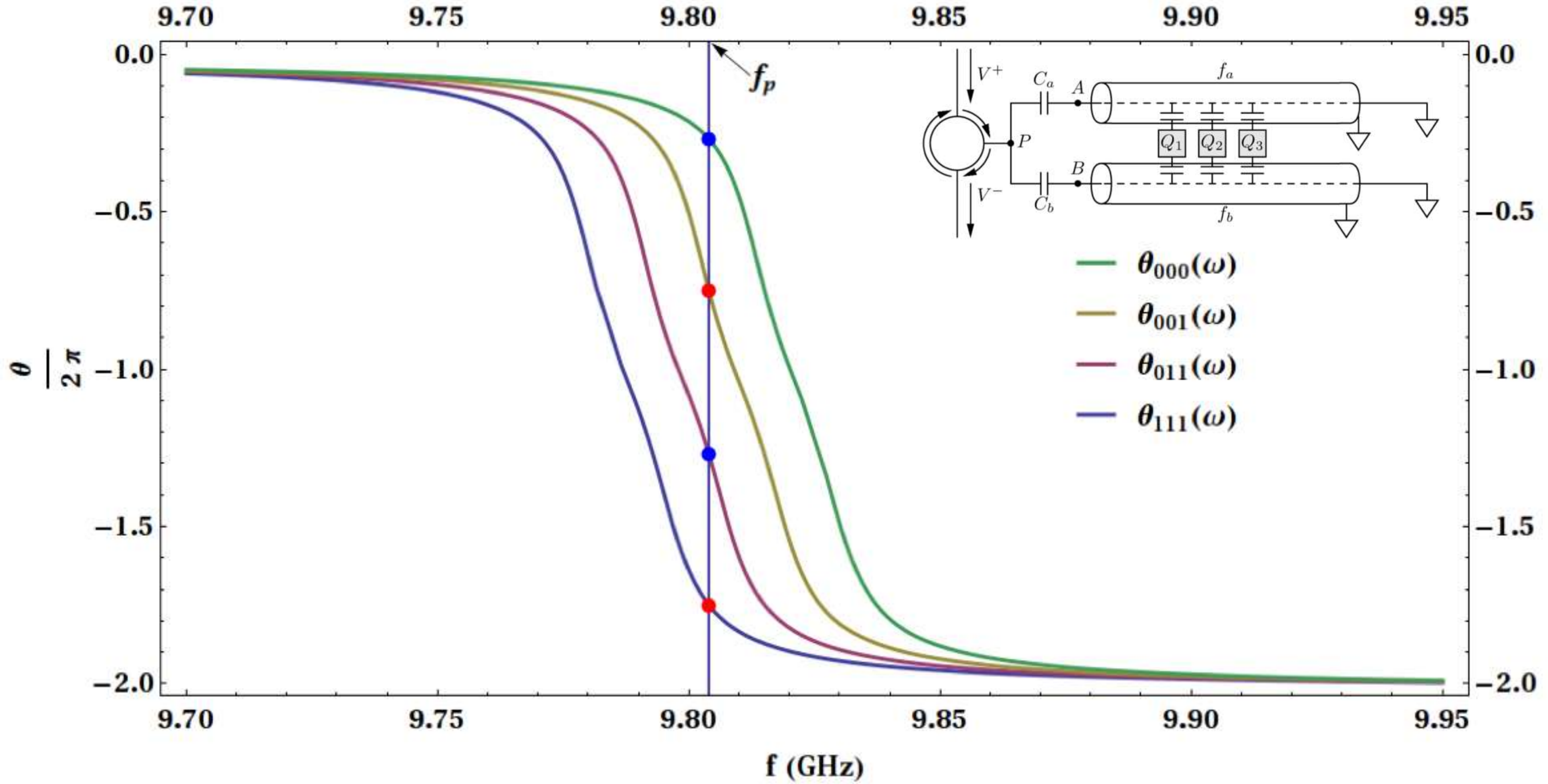


Figure 3. Solution for realistic values $\omega_a = 2\pi(9.99 \text{ GHz})$, $\omega_b = 2\pi(10.01 \text{ GHz})$, $C_a = C_b = 10\,fF$; with $\omega_p = 2\pi(9.804\,GHz)$, $\chi = 5.77\,MHz$, giving $\Delta\theta = 172.9°$. Blue points correspond to even states whereas red points correspond to odd states. Vertical blue line shows the probe frequency $f_p = 9.804\,GHz$.

θ is the same for all even states (mod 2π)
θ is the same for all odd states (mod 2π)

$\theta_{even} \neq \theta_{odd}$

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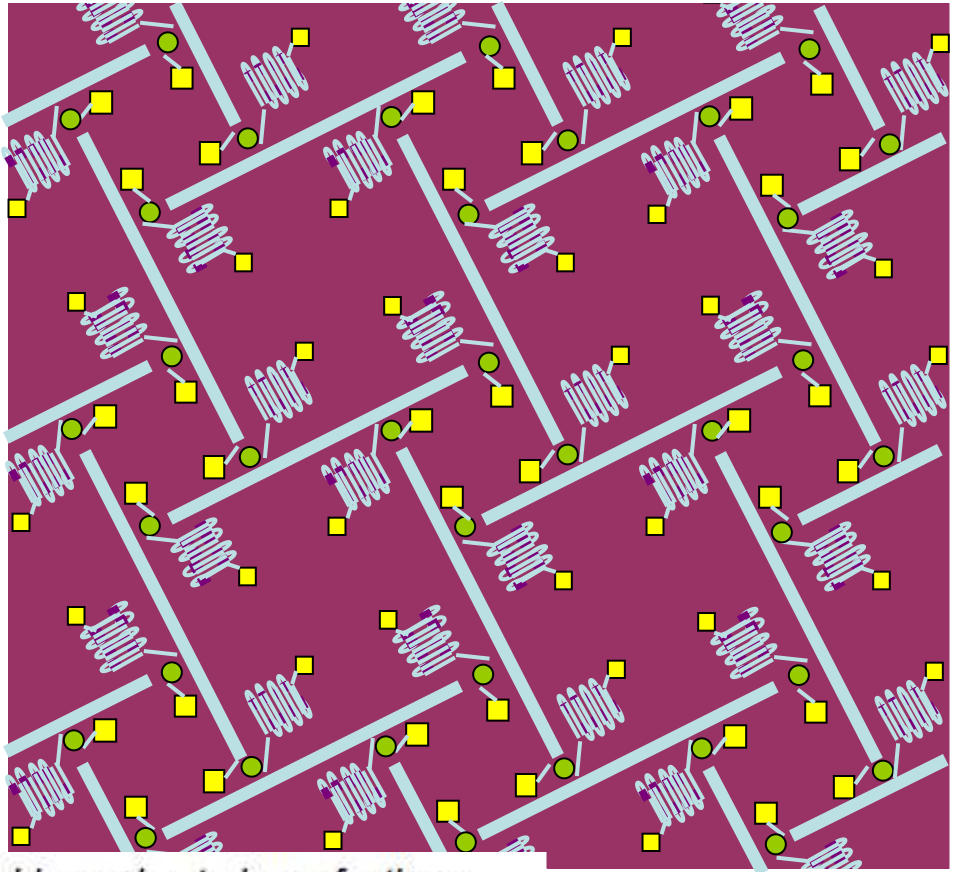

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“No power is required to perform computation.”
CH Bennett

“Quantum computers can operate autonomously.”
N Margolus

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David DiVincenzo

26.9.2012

ETHZ

-- positions available!
-- 2-week lecture school, Feb. 2013: www.iff-springschool.de

Prospects for Superconducting Qubits

David DiVincenzo

7.10.2012

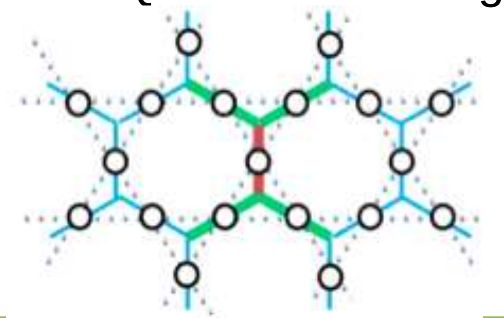
LCN

-- 2-week lecture school,
Feb. 2013:
www.iff-springschool.de

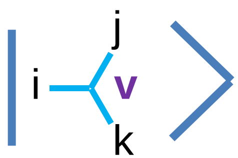
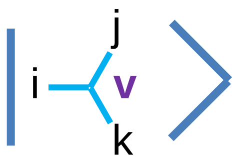
Levin-Wen Models

$$H = -\sum_v Q_v - \sum_p B_p$$

Trivalent lattice: Qubits live on edges



Vertex Operator

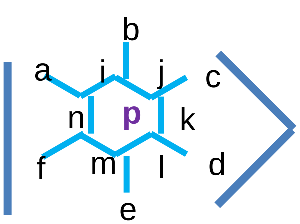
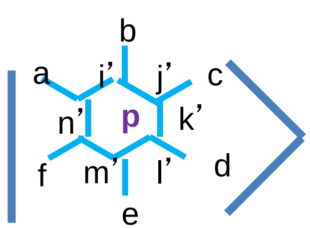
Q_v  $= \delta_{ijk}$ 

“Doubled Fibonacci” Model

$$\delta_{100} = \delta_{010} = \delta_{001} = 0$$

All other $\delta_{ijk} = 1$

Plaquette Operator

$B^s_{\mathbf{p}}$  $= \sum_{i'j'k'l'm'n'} B^{s,i'j'k'l'm'n'}_{\mathbf{p},ijklmn}(abcdef)$ 

Horrible 12 spin interaction!

$$B_{\mathbf{p}} = \frac{B^0_{\mathbf{p}} + \varphi B^1_{\mathbf{p}}}{\sqrt{1+\varphi^2}}$$

$$B^{s,i'j'k'l'm'n'}_{\mathbf{p},ijklmn}(abcdef) = F^{ani}_{\mathrm{s}i'n'} F^{bij}_{sj'i'} F^{cjk}_{sk'j'} F^{dkl}_{sl'k'} F^{elm}_{sm'l'} F^{fmn}_{sn'm'}$$

"Fibonacci" Levin-Wen Model

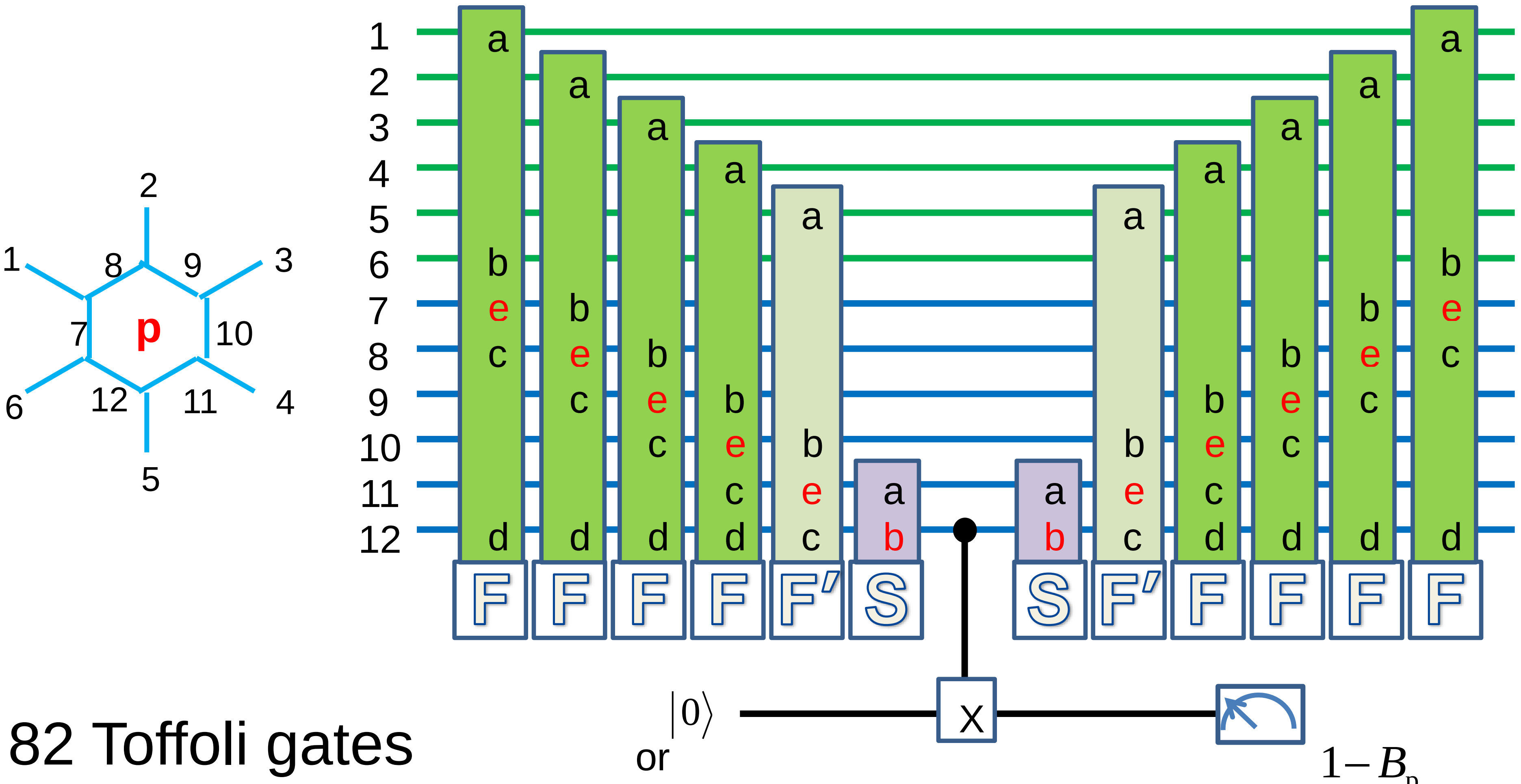
- **Excitations are Fibonacci anyons:** Universal quantum computation can be carried out purely by braiding.

- **Active approach:** Ground states of Fibonacci Levin-Wen model can be used as a quantum code (the Fibonacci code). $\boldsymbol{Q_v}$ and $\boldsymbol{B_p}$ are stabilizers which are measured to diagnose errors.

Koenig, Kuperberg, Reichardt, Ann. of Phys. (2010).

Question: How hard is it to measure Q_v and B_p?

Quantum Circuit for Measuring B_p

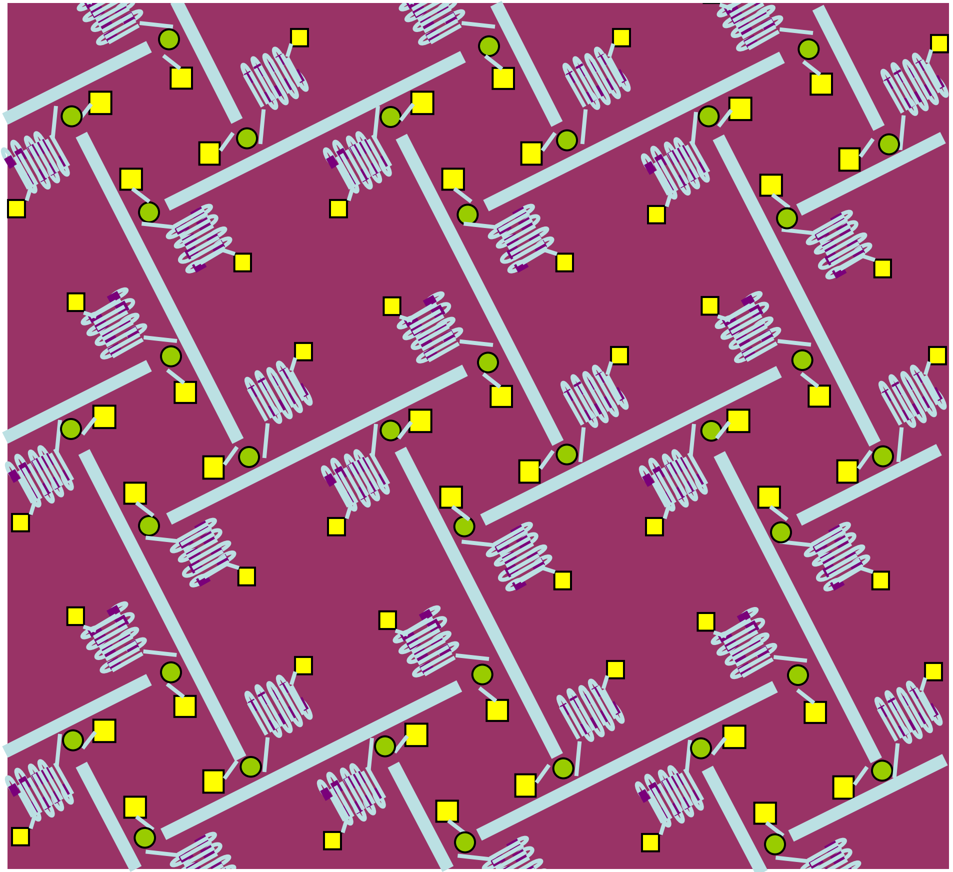


82 Toffoli gates
43 CNOT gates
26 Single Qubit gates

or
20 n-qubit Toffoli gates
10 CNOT gates
24 Single Qubit gates

Conclusion: quantum error correction in your future

- Original insights still being played out
- Maybe a good evolutionary path to quantum computer hardware



Concept (IBM) of surface code fabric with Superconducting qubits and coupling resonators

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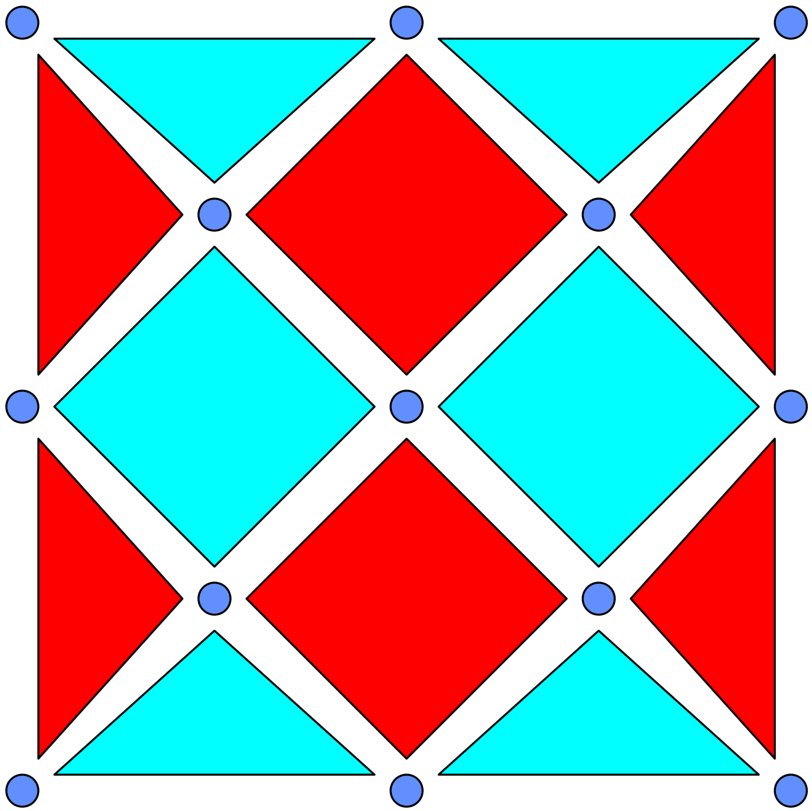
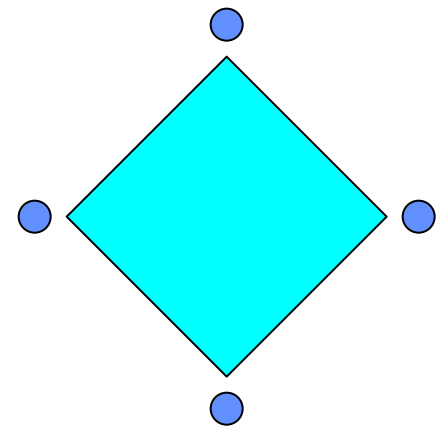
Another view of the 2D Surface Code

With these 13 qubits, one gets a standard code that will correct for one error:



4-qubit QND parity measurement:



Red diamond: the same in the conjugate basis

S. Bravyi and A. Yu. Kitaev, "Quantum codes on a lattice with boundary," Quantum Computers and Computing **2**, 43-48 (2001).
M. H. Freedman and D. A. Meyer, "Projective plane and planar quantum codes," Found. Comp. Math. **1**, 325 (2001)

Observations:

Calculated fault tolerant threshold:

$p \approx 0.7\%$

Now $p \approx 1\ \%$, according to Wang, Fowler, Hollenberg, Phys. Rev. A 83, 020302(R) (2011)

Crosstalk assumed "very small", not analyzed

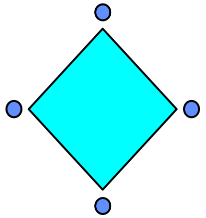
Residual errors decrease exponentially with lattice size

Gates: CNOT only (can be CPHASE), no one qubit gates

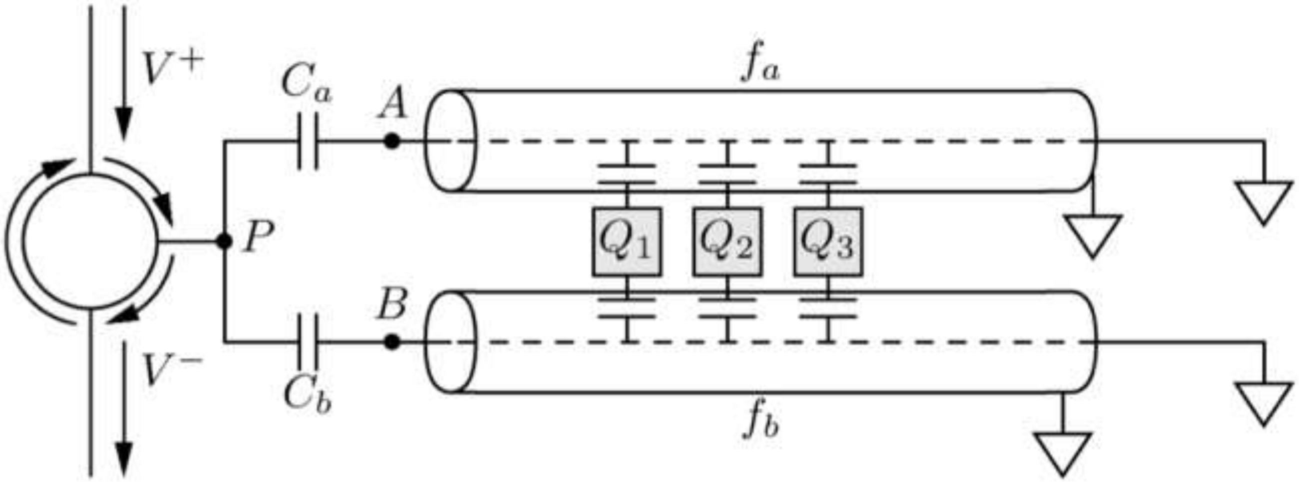
If measurements slow: more ancilla qubits needed, no threshold penalty

NB: Error threshold for 4-qubit Parity QND measurement is around

$2\% < p < 12\%$



A two-resonator device for measuring the parity of three qubits:



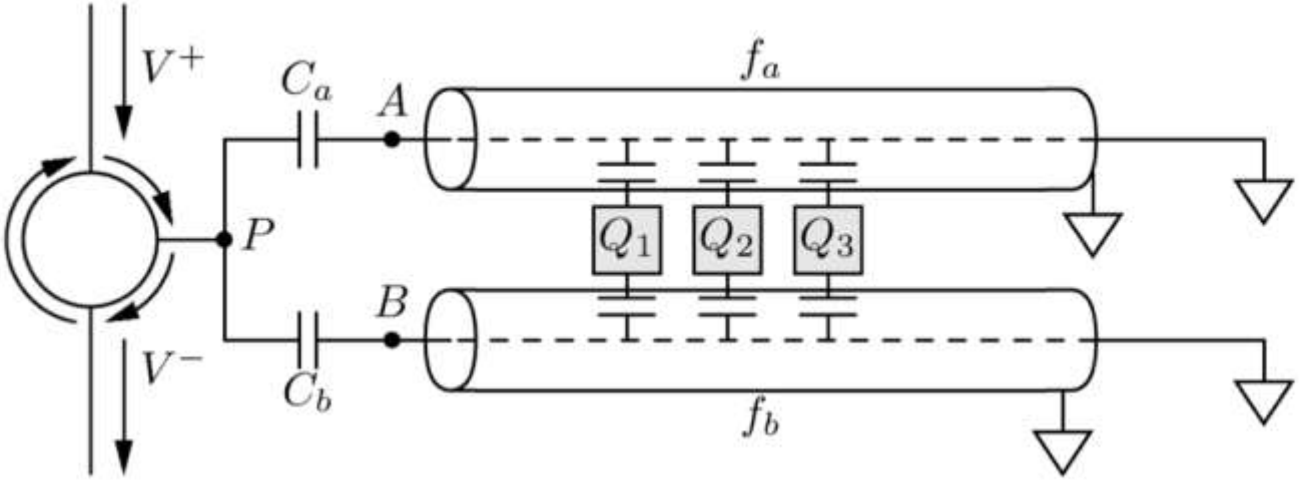
$$\omega_{r,a} = \omega_a + \sum_{j=1}^{3} (-1)^{s_j} \chi_j .$$

$\chi = g^2/\Delta$

s_1, s_2, s_3 are the states of the three qubits (0,1)

χ_i is dispersive shift parameter

Dispersive coupling is the same for each qubit and the same on both resonators (a and b)



$$Z_A(\omega) = iZ_0 \tan\left(\frac{\pi}{2}\frac{\omega}{\omega_{r,a}}\right)$$

Wave impedance "looking into" port A (transmission line theory)

$$r(\omega) = \frac{V^-(\omega)}{V^+(\omega)} = \frac{Z_P(\omega) - Z_0}{Z_P(\omega) + Z_0}$$

(Z_0=50Ω)

Reflection coefficient of full structure

NB $|r| = 1$