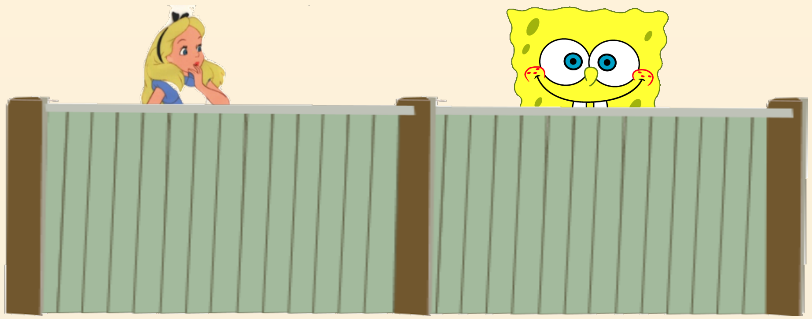

Everything You Always Wanted to Know About LOCC

(But Were Afraid to Ask)

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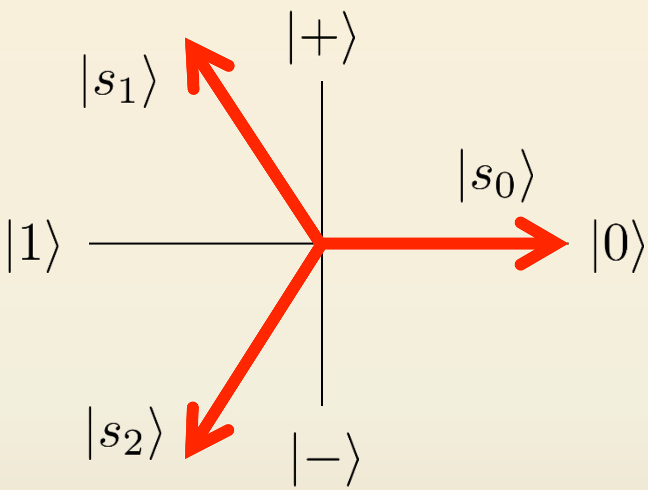
Background & Motivation – The Need for Precise Definitions

- 1991 - Peres and Wootters introduce the LOCC setting:[1]

$$|\psi_1\rangle = |s_0\rangle_A \otimes |s_0\rangle_B$$
$$|\psi_2\rangle = |s_1\rangle_A \otimes |s_1\rangle_B$$
$$|\psi_3\rangle = |s_2\rangle_A \otimes |s_2\rangle_B$$

"Double Trine Ensemble"



Goal: Alice and Bob attempt to identify their shared state.

Results: (1) Alice and Bob acting separately apparently reduces success probability compared to joint action.
(2) An adaptive, multiple-round strategy seems optimal.

Background & Motivation – The Need for Precise Definitions

- 1999 - Bennett *et al.* show "non-locality without entanglement"[2].

- States can be perfectly distinguished by <u>product state</u> projectors.
- However the states *cannot* be distinguished perfectly by LOCC.
  - Cannot be distinguished "asymptotocially", or with ϵ error probability [3,4].

Alice

$|0\rangle$ $|1\rangle$ $|2\rangle$

Bob

$$|\psi_0\rangle = |1\rangle \otimes |1\rangle$$

$$|\psi_1^{\pm}\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes (|1\rangle \pm |2\rangle)$$

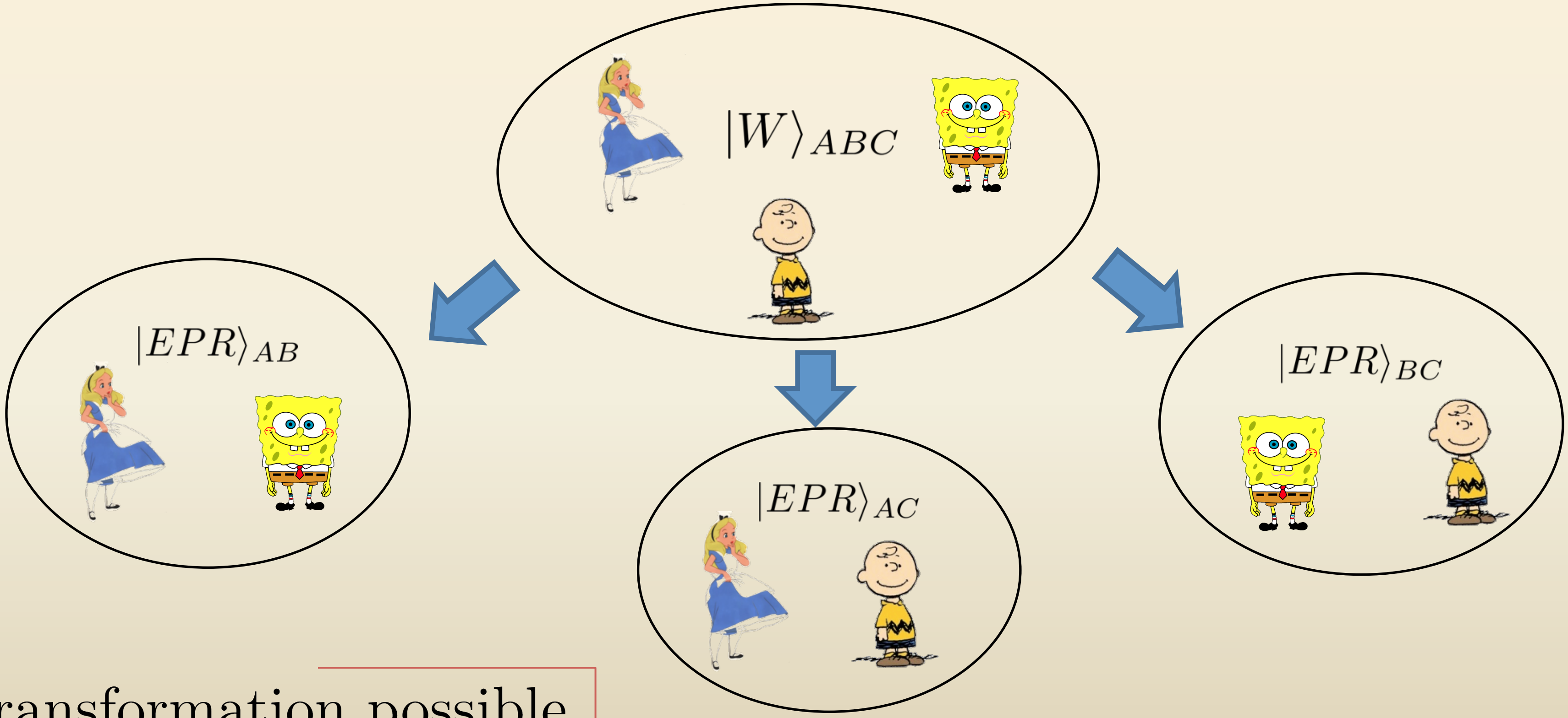
$$|\psi_2^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle) \otimes |0\rangle$$

$$|\psi_3^{\pm}\rangle = \frac{1}{\sqrt{2}}|2\rangle \otimes (|0\rangle \pm |1\rangle)$$

$$|\psi_4^{\pm}\rangle = \frac{1}{\sqrt{2}}(|1\rangle \pm |2\rangle) \otimes |2\rangle$$

Background & Motivation – The Need for Precise Definitions

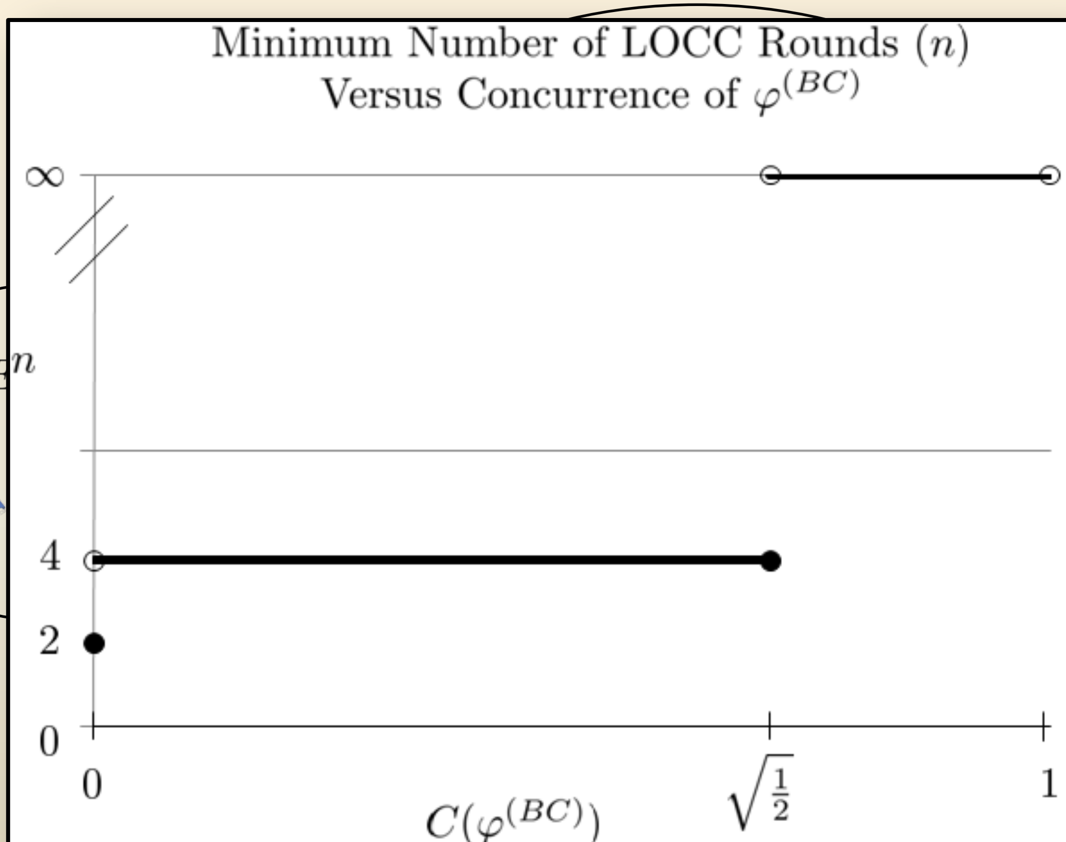
- 2011 - The task of tripartite "random distillation" is shown to be achievable by LOCC *only* asyomptotically[5].



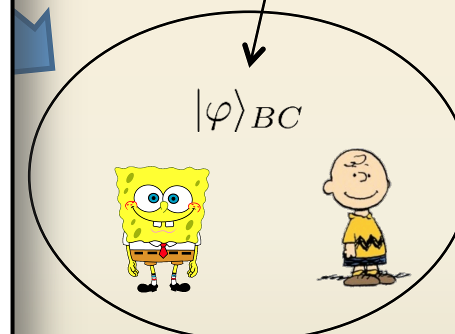
Transformation possible with probability $1-\epsilon$ iff $\epsilon > 0$

Asymptotic in success probability

Background & Motivation – The Need for Precise Definitions



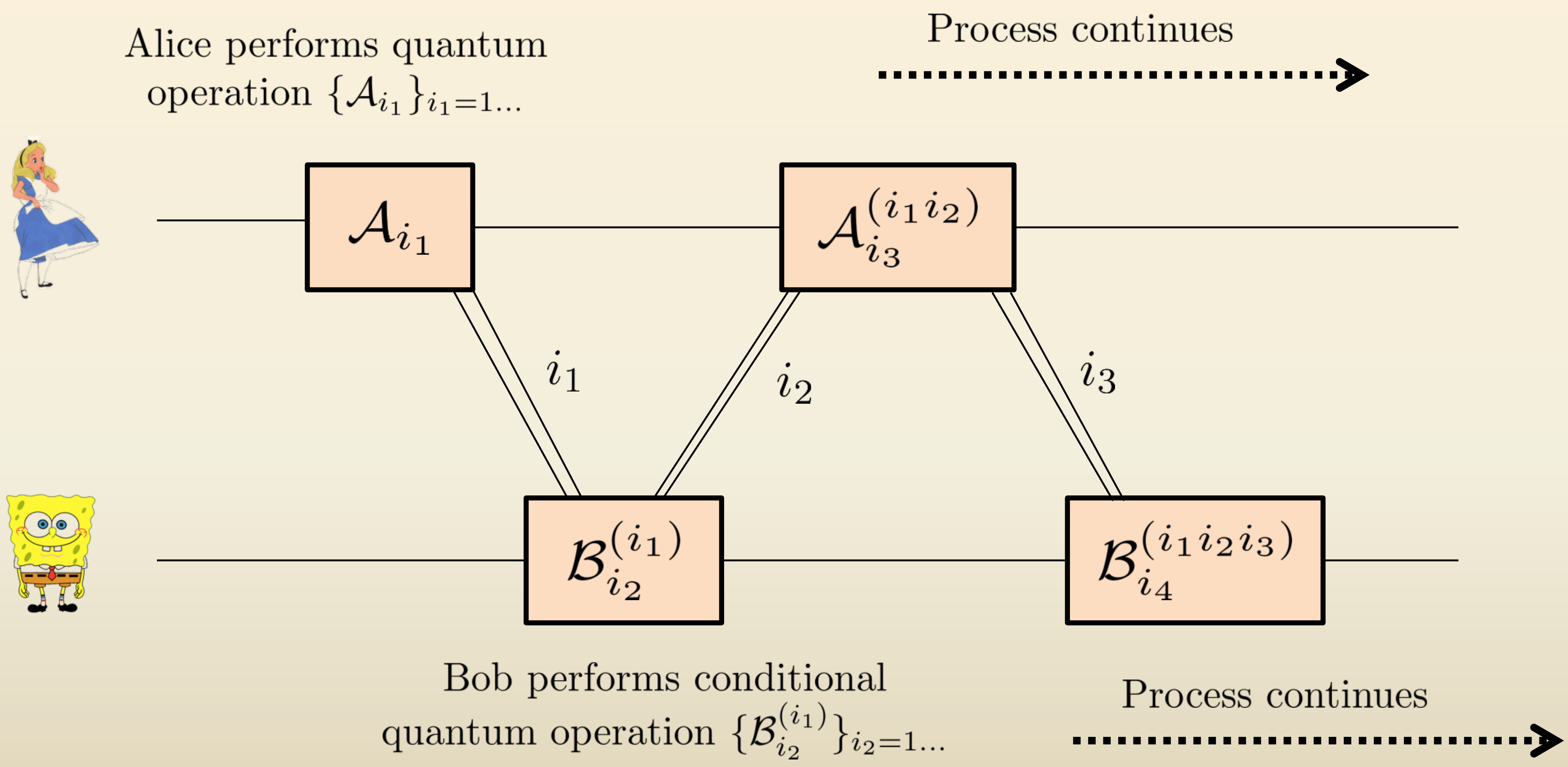
- Modify the task by reducing the distilled entanglement[6].



Asymptotic
in round number

Defining the Class of LOCC

- A general LOCC protocol:



Defining the Class of LOCC

- We identify LOCC protocols as *quantum instruments*, or sequences of CP maps $\mathfrak{J} = (\mathcal{E}_1, \mathcal{E}_2, ...)$ built up in this manner.

- LOCC_r denotes the set of all instruments realizable in r rounds.

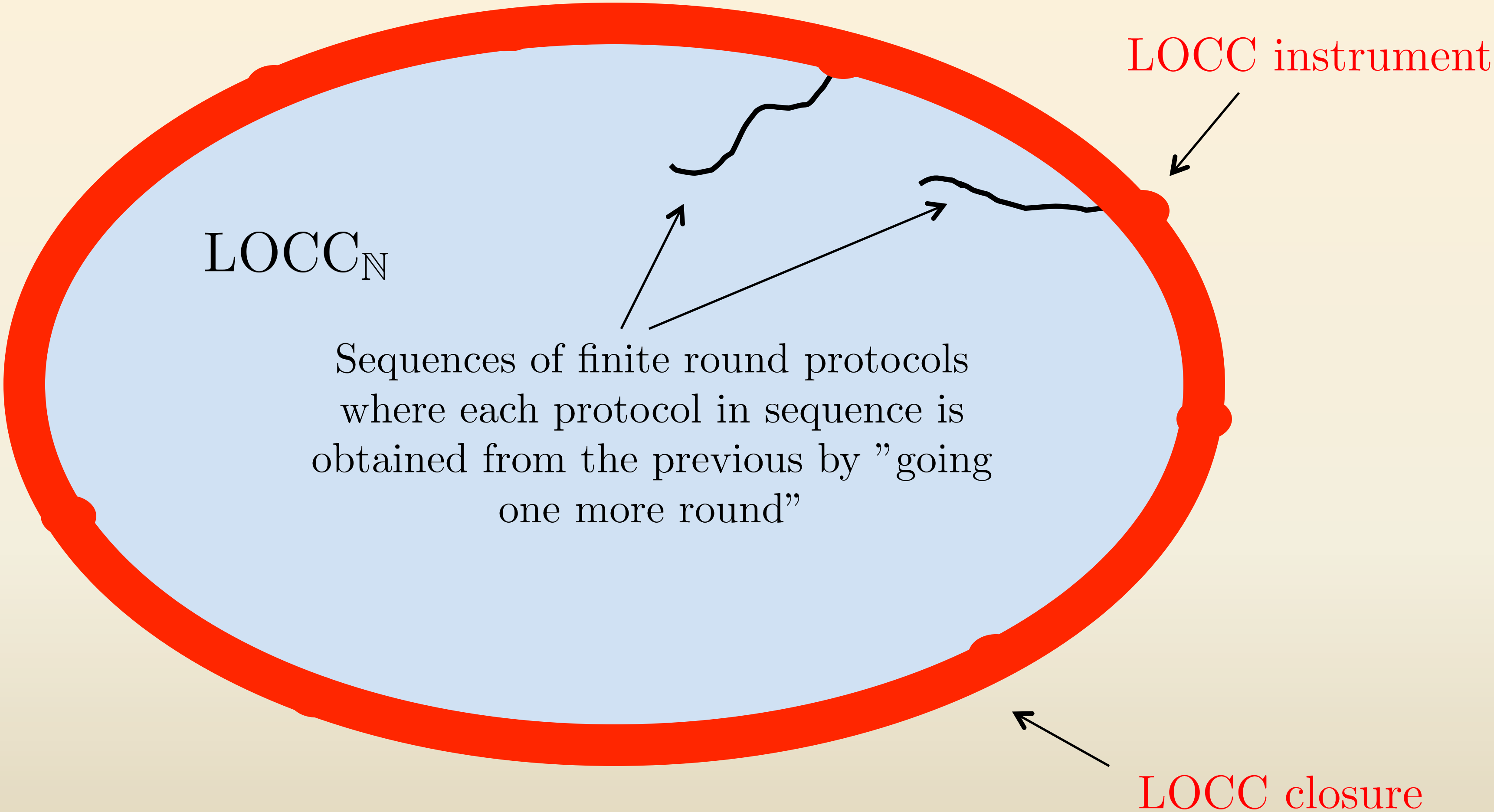
- $\text{LOCC}_{\mathbb{N}} := \cup_r \text{LOCC}_r$. ⟵ Bounded-round protocols

- A distance between two instruments $\mathfrak{J} = (\mathcal{E}_1, ..., \mathcal{E}_k)$ and $\tilde{\mathfrak{J}} = (\mathcal{F}_1, ..., \mathcal{F}_k)$ is given by $||\mathfrak{J}, \tilde{\mathfrak{J}}||_\diamond := \sum_{j=1}^{k} ||\mathcal{E}_j - \mathcal{F}_j||_\diamond$.

- LOCC is the union of $\text{LOCC}_{\mathbb{N}}$ and the limit points of all unbounded-round protocols (that converge).

- $\overline{\text{LOCC}_{\mathbb{N}}}$ denotes the closure of $\text{LOCC}_{\mathbb{N}}$.

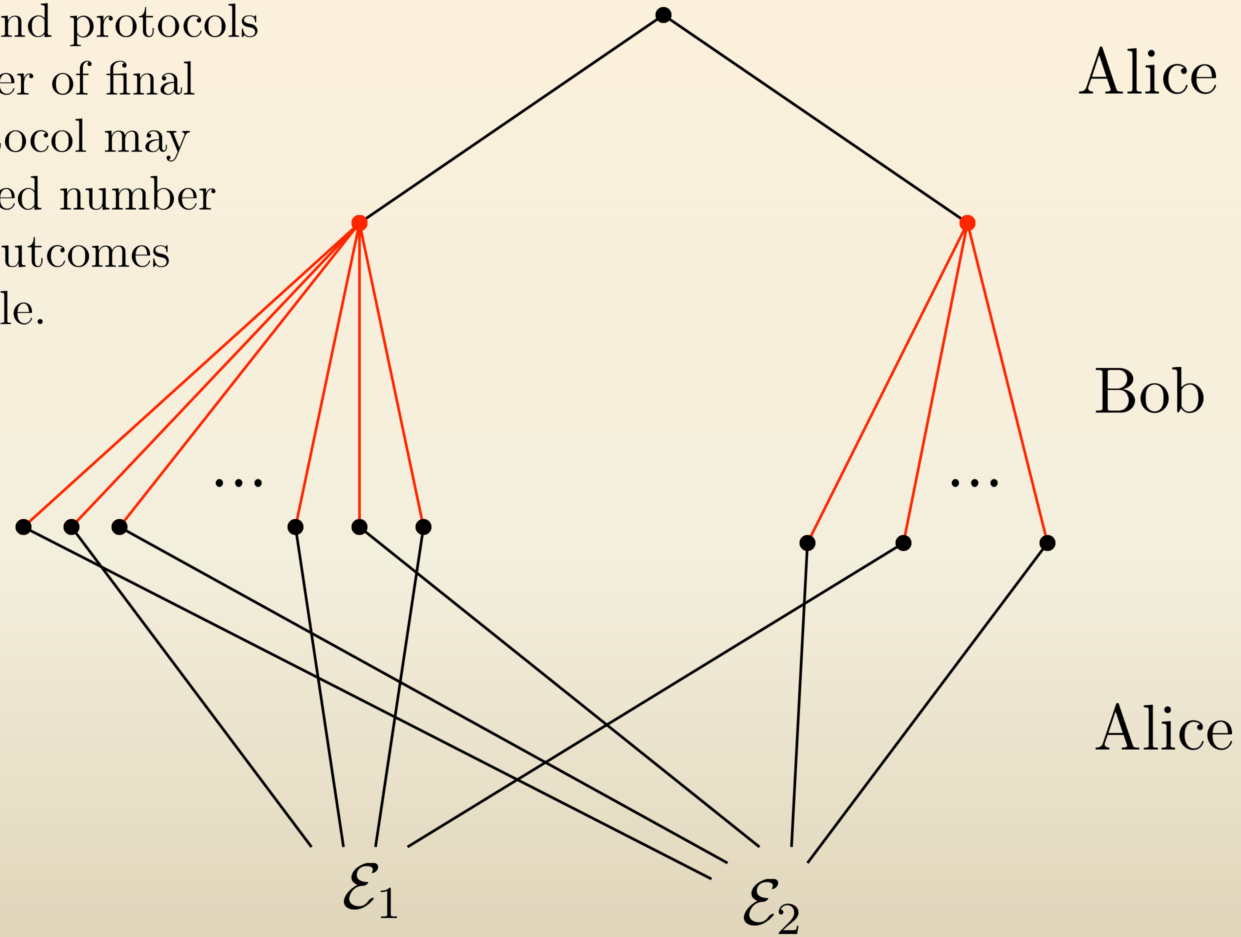
LOCC Instruments



What classes of LOCC protocols are closed?

Why Bounded-Round Closure is Perhaps not Obvious

Even for bounded-round protocols with a finite number of final outcomes, the protocol may involve an unbounded number of measurement outcomes in the middle.

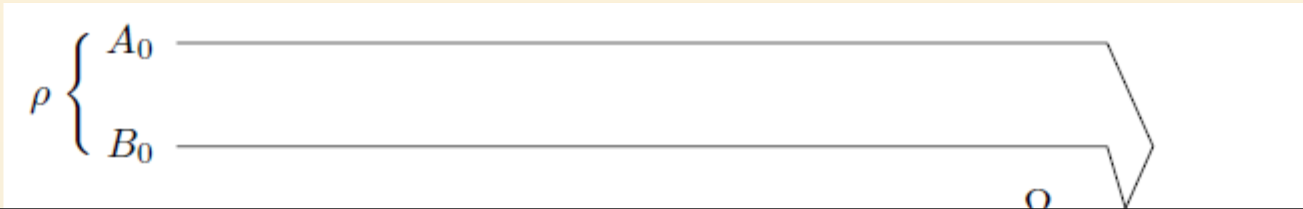


Bounded-Round Compactness

Lemma. For an N-partite system of total dimension D, suppose that $\mathfrak{J} = (\mathcal{E}_1, \dots, \mathcal{E}_m)$ is an instrument in LOCC_r.

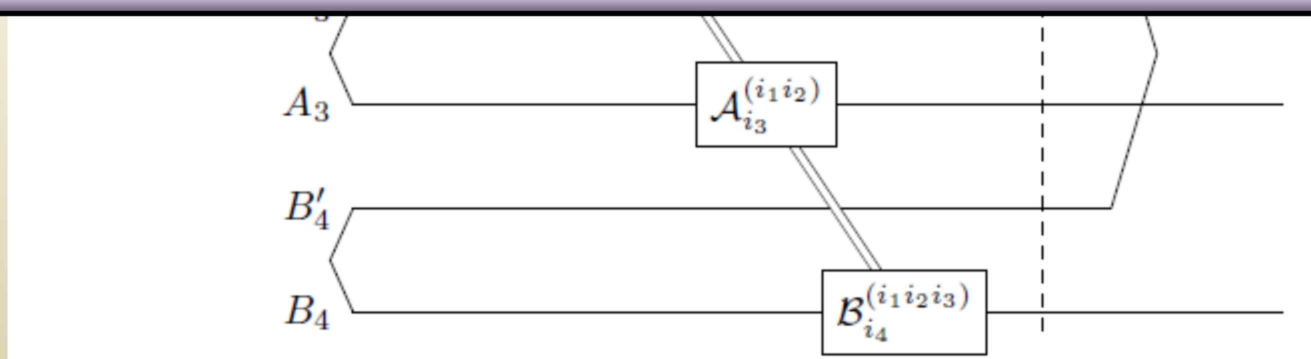
Then there exists an r-round protocol that implements $(\mathcal{E}_1, \dots, \mathcal{E}_m)$ such that each instrument in round $1 \le l \le r$ consists of no more than $D^{4(r-l+1)}$ maps.

Effective "Choi Matrix" Ω for LOCC Protocol

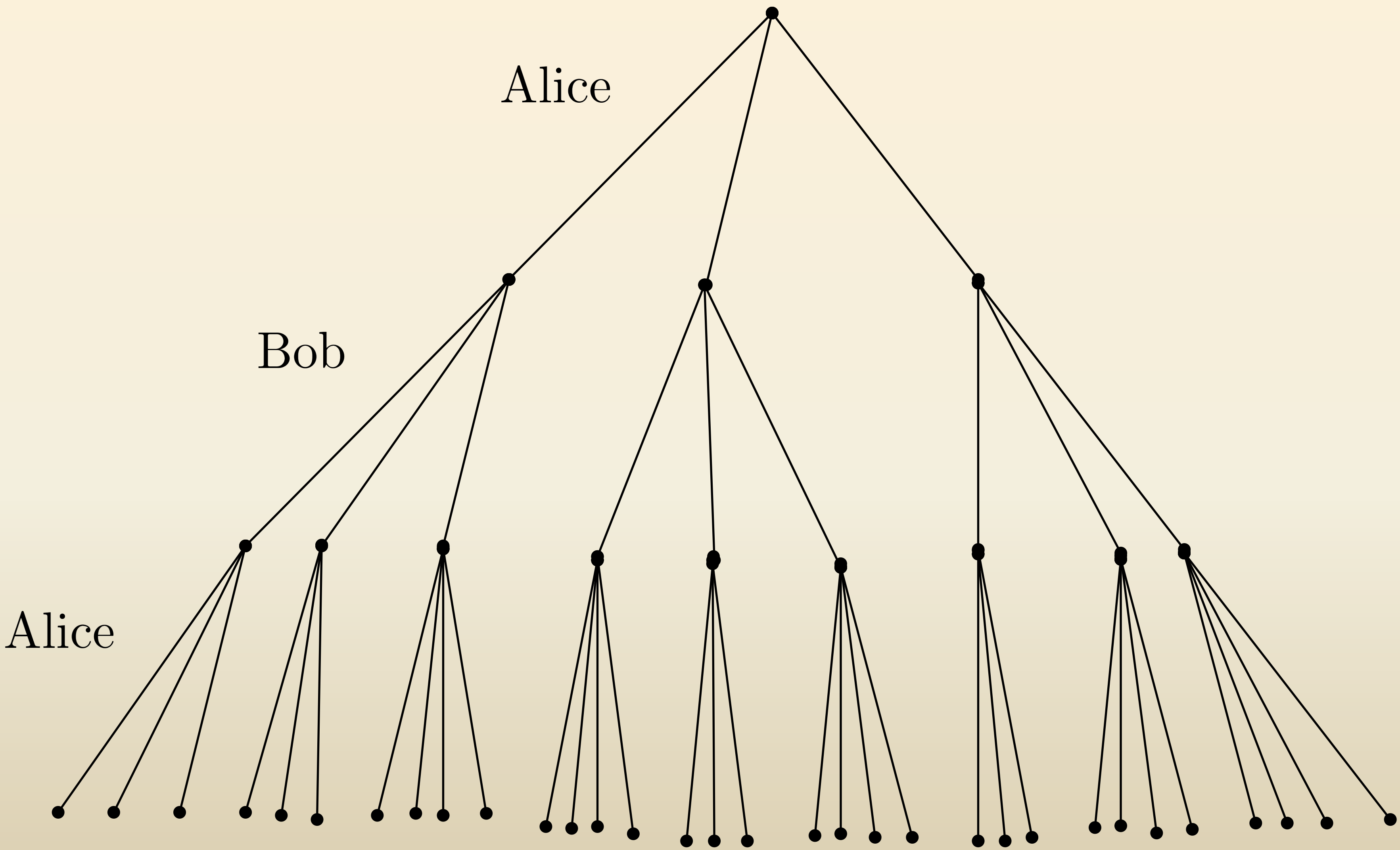


Carathéodory's Theorem:

Let S be a subset of $\mathbb{R}^n$ and $conv(S)$ its convex hull. Then any $x \in conv(S)$ can be expressed as a convex combination of at most $n+1$ elements of S.



Cleaning Up the LOCC Tree



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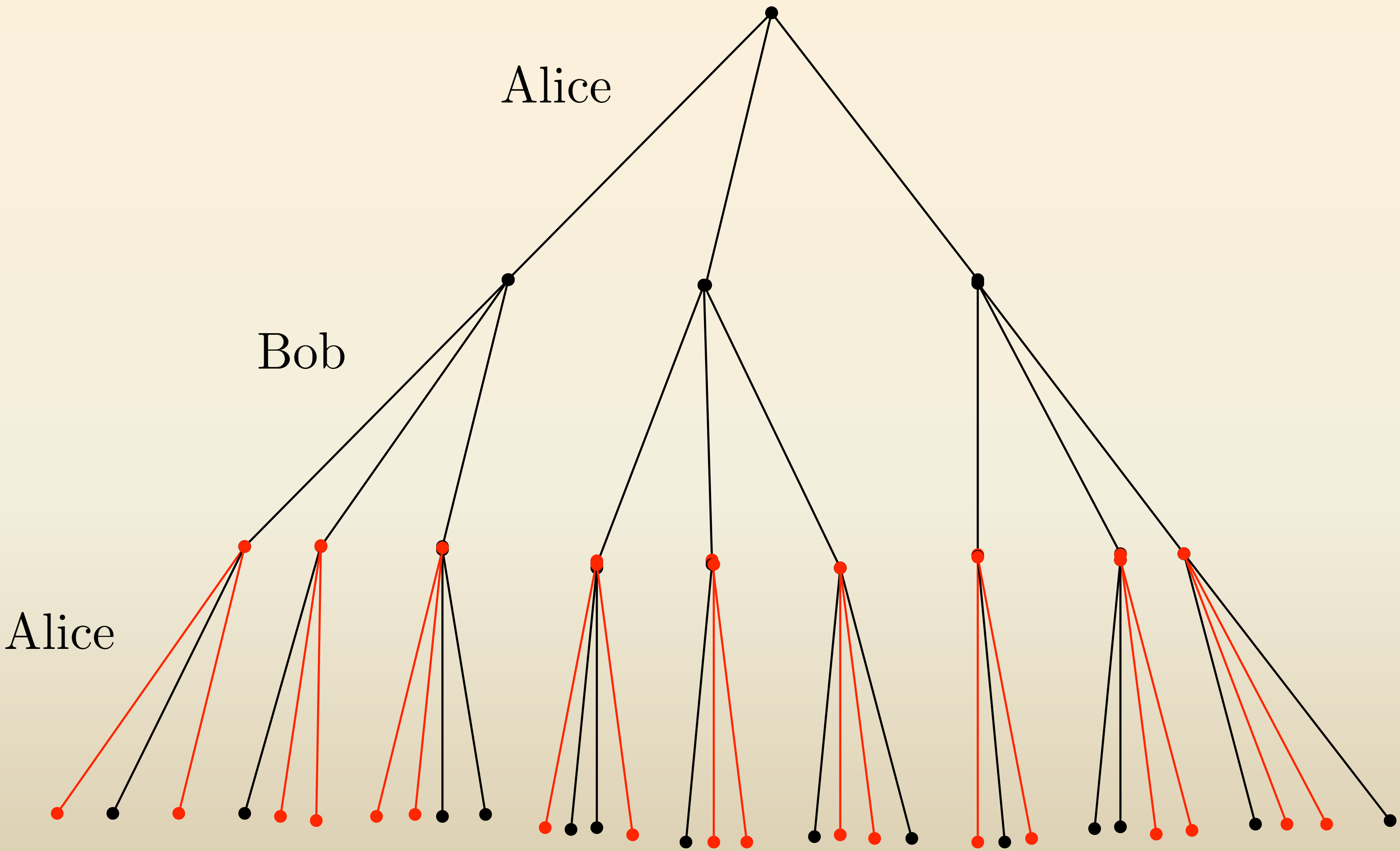

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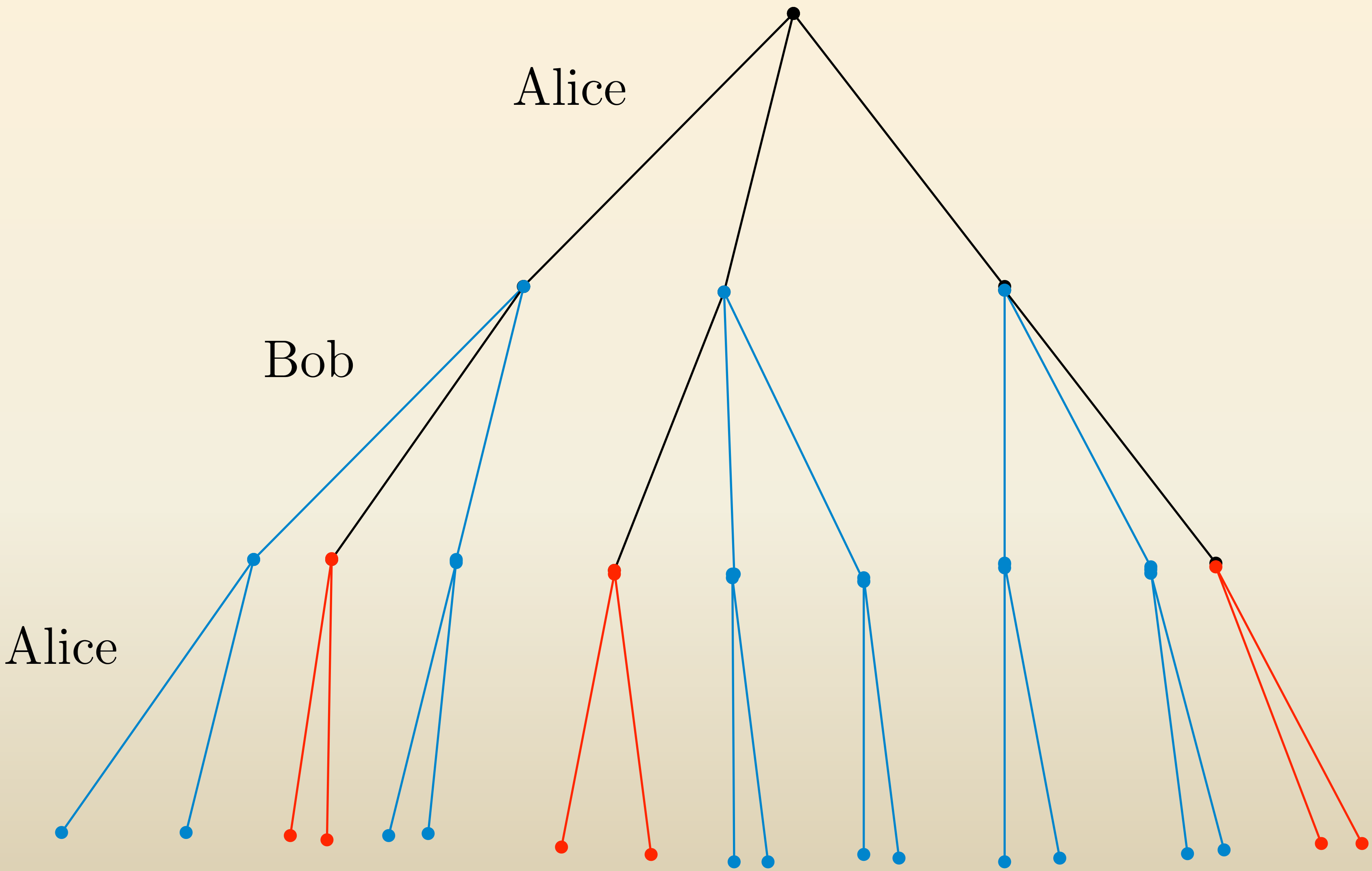

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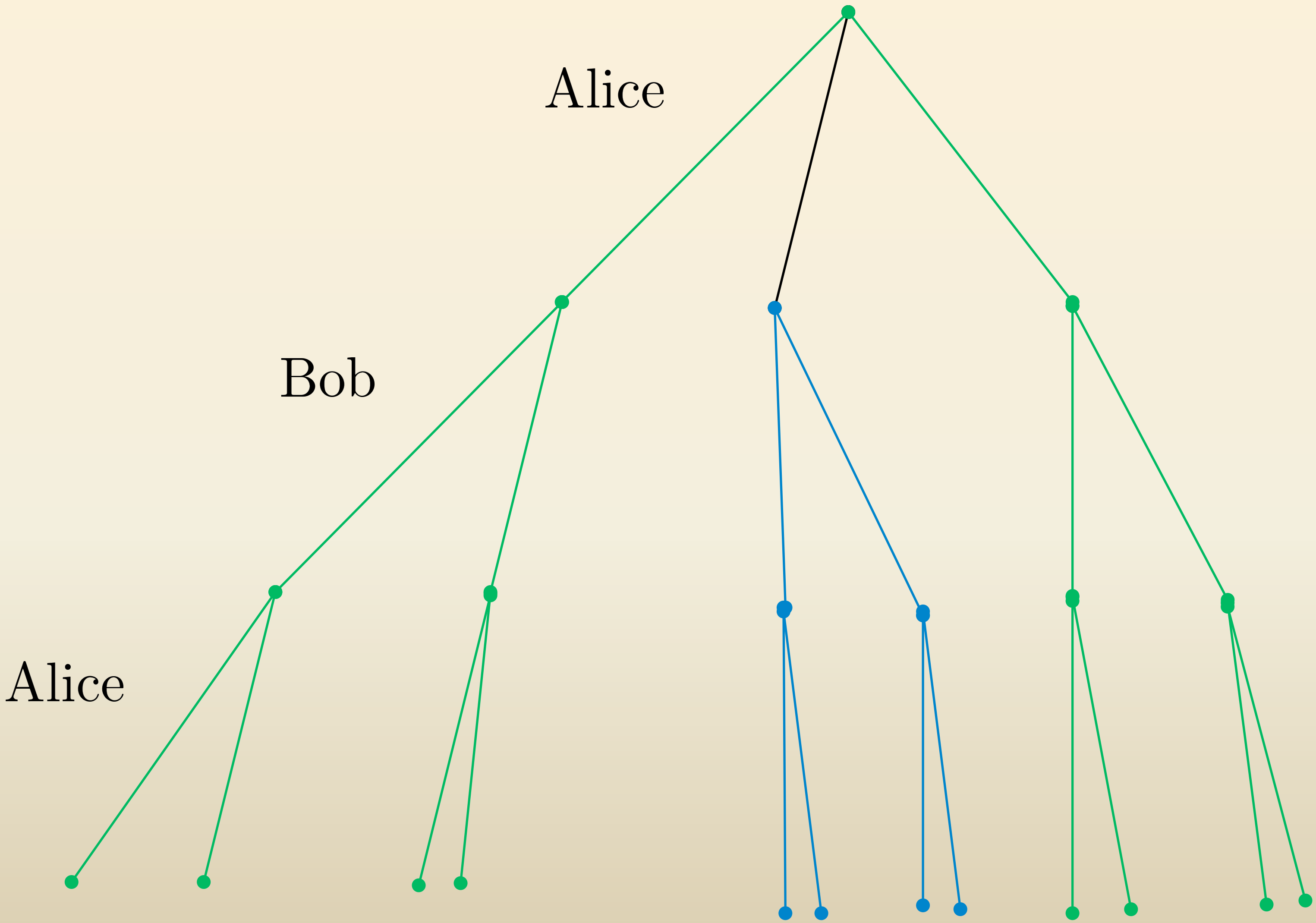


Bipartite Sequences with Unbounded Rounds

- Borrow from the *tripartite* random distillation scheme.

$$M_0 = \begin{pmatrix} \sqrt{1-\epsilon} & 0 \\ 0 & 1 \end{pmatrix} \qquad M_1 = \begin{pmatrix} \sqrt{\epsilon} & 0 \\ 0 & 0 \end{pmatrix}$$

- Alice and Bob both measure $\{M_0, M_1\}$.

- If either of them obtains outcome "0," they halt. Otherwise, they repeat the measurement.

- They repeat this for ν iterations.

- There are four possible outcomes per iteration, and we coarse-grain over all the interations:

Two parameters: ν and ϵ →

$$\mathfrak{J}_\nu(\epsilon) = (\mathcal{E}_{\nu 00}, \mathcal{E}_{\nu 01}, \mathcal{E}_{\nu 10}, \mathcal{E}_{\nu 11}).$$

Bipartite Sequences with Unbounded Rounds

$$\mathfrak{J}_\nu(\epsilon) = (\mathcal{E}_{\nu 00}, \mathcal{E}_{\nu 01}, \mathcal{E}_{\nu 10}, \mathcal{E}_{\nu 11}).$$

- Letting $\nu \to \infty$ gives a sequence (indexed by ϵ) of unbounded round protocols.
- Letting $\epsilon \to 0$ gives the limit instrument of this sequence.
- The three element limit instrument:

$$\mathcal{E}_{00}(\rho) = |11\rangle\langle 11|\rho|11\rangle\langle 11|,$$

$$\mathcal{E}_{01}(\rho) = \sum_{i=1}^{2}(T_i \otimes |0\rangle\langle 0|)\rho(T_i^\dagger \otimes |0\rangle\langle 0|),$$

$$\mathcal{E}_{10}(\rho) = \sum_{i=1}^{2}(|0\rangle\langle 0| \otimes T_i)\rho(|0\rangle\langle 0| \otimes T_i^\dagger)$$

$$T_1 = \sqrt{1/3}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$T_2 = \sqrt{1/3}\begin{pmatrix} \sqrt{1/2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$$

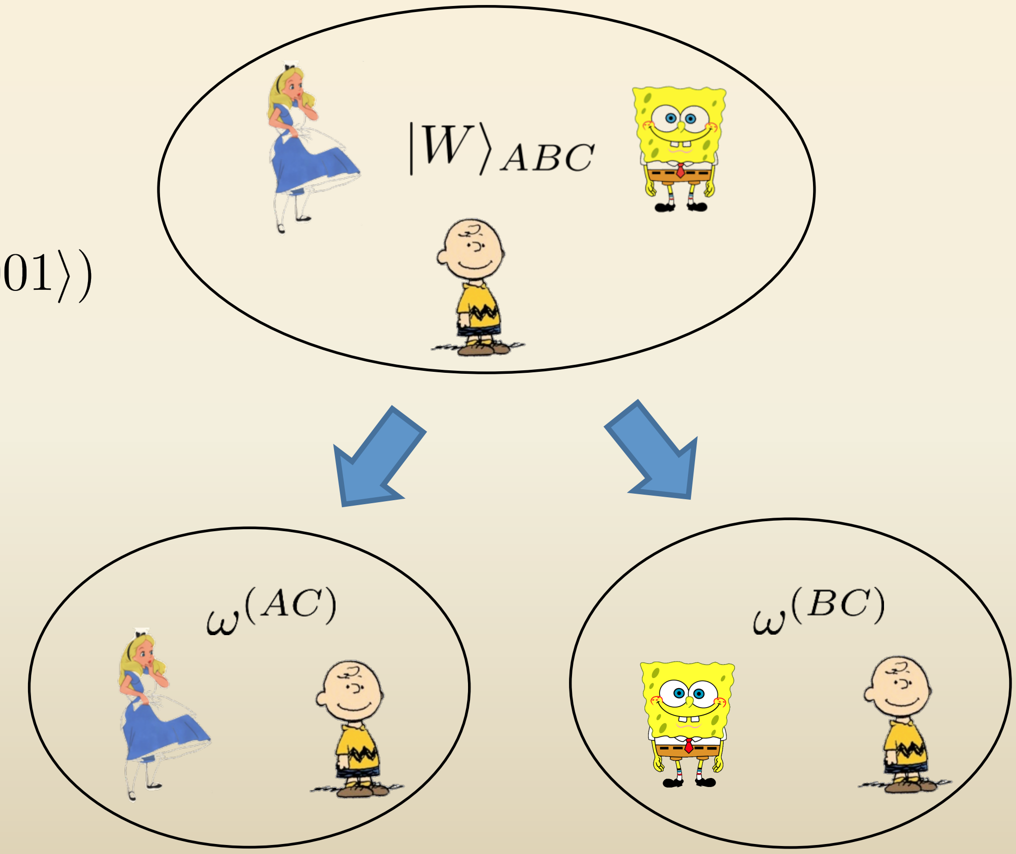
Bipartite Sequences with Unbounded Rounds

- We embed the protocol into tripartite system where Alice and Bob are entangled with Charlie.

- Charlie acts trivially.

$$|W\rangle = \sqrt{1/3}(|100\rangle + |010\rangle + |001\rangle)$$

$$\omega = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/3 & 4/9 & 0 \\ 0 & 4/9 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



This transformation is impossible!

Conclusions and Open Questions

• The set of n-outcome instruments in LOCC_r is compact.

• For bipartite systems:

$$\mathrm{LOCC} \subset \overline{\mathrm{LOCC}} \subset \mathrm{SEP}$$

all proper inclusions.

• If we restrict attention only to POVMs, might it be that $\mathrm{LOCC} = \overline{\mathrm{LOCC}}$?

• For LOCC protocols with unbounded rounds, does there exist a relationship between round number and rate of convergence?

Thank You!

References

1 – Peres & Wootters – *Phys. Rev. Lett.* **66,** 1119 (1991).

2 – Bennett *et al.* – *Phys. Rev. A* **59**, 1070 (1999).

3 – Kleinmann *et al.* – *Phys. Rev. A* **84**, 042326 (2011).

4 – Childs *et al.* – arXiv:1206.5822 (2012).

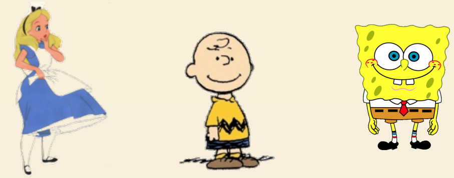
5 – Chitambar *et al.* – *Phys. Rev. Lett.* **108**, 240504 (2012).

6 – Chitambar – Phys. Rev. Lett. **107**, 190502 (2011).

Random Concurrence Distillation

$$|W'\rangle = \sqrt{x_0}|000\rangle + \sqrt{x_A}|100\rangle + \sqrt{x_B}|010\rangle + \sqrt{x_C}|001\rangle$$

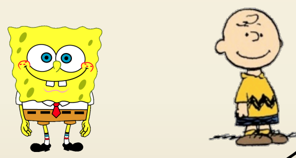
General W-Class State



$(x_B \geq x_C)$

$|\varphi_i^{(AB)}\rangle$

$|\varphi_i^{(BC)}\rangle$



- Optimize:

$$< C_{tot} >= \sum_i p_i^{(AC)} C(\varphi_i^{(AC)}) + \sum_i p_i^{(BC)} C(\varphi_i^{(BC)})$$

- **Result:**

$$< C_{tot} > \leq \mathcal{C}(W') := 2\sqrt{x_A x_B} + x_C\sqrt{\frac{2}{3}\frac{x_A}{x_B}}$$

Entanglement Monotone!

Random Concurrence Distillation

- The limit instrument $(\mathcal{E}_{00}, \mathcal{E}_{01}, \mathcal{E}_{10})$ transforms:

$$|W\rangle\langle W| \longrightarrow \begin{cases} \omega^{(AC)} \otimes |0\rangle\langle 0|^{(B)} & \text{w. prob. } 1/2, \\ |0\rangle\langle 0|^{(A)} \otimes \omega^{(BC)} & \text{w. prob. } 1/2, \end{cases} \qquad \omega = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/3 & 4/9 & 0 \\ 0 & 4/9 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- For $\rho \in$ Mixed W-class,

$$\hat{\mathcal{C}}(\rho) = \min_{p_i, |\phi_i\rangle} \sum_{p_i} p_i \, \mathcal{C}(\phi_i) \quad \text{is an entanglement monotone.}$$

- Initial concurrence: $\hat{\mathcal{C}}(|W\rangle\langle W|) = 8/9$.

- Final concurrence: $\hat{\mathcal{C}}(\omega) = 8/9$.

- Therefore, the transformation is impossible since $\hat{\mathcal{C}}$ strictly decreases upon first non-trivial measurement.