Symmetry protection of measurement-based quantum computation in ground states

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The two-dimensional cluster state, a universal resource for measurement-based quantum computation, is also the gapped ground state of a short-ranged Hamiltonian. Here, we examine the effect of perturbations to this Hamiltonian. We prove that, provided the perturbation is sufficiently small and respects a certain symmetry, the perturbed ground state remains a universal resource. We do this by characterizing the operation of an adaptive measurement protocol throughout a suitable symmetry-protected quantum phase, relying on generic properties of the phase rather than any analytic control over the ground state.

Introduction

A quantum computer relies on quantum entanglement to achieve computational speedups. In the traditional, circuit-based model for quantum computation, the required entanglement is built up throughout the course of the computation through application of entangling gates coupling two or more qubits at a time. Alternatively, in the model of *measurement-based quantum computation* (MBQC) [1, 2], universal quantum computation is achieved solely through single-particle operations (specifically, single-particle measurements) on a fixed entangled resource state, independent of the quantum algorithm being performed.

Since the initial discovery that the 2-D cluster state is a universal resource for MBQC [1], much effort has been devoted to characterizing other universal resource states. Many of the universal resource states so far identified [1, 3–6] have been *projected entangled pair states* (PEPS) [7] of small bond dimension. The tensor network structure of these states facilitates the analysis of measurements, which might otherwise be an intractable problem. Another advantage of such states is that under appropriate conditions [8], they are unique (possibly gapped) ground states of local frustration-free Hamiltonians on spin lattices. This suggests a method of constructing the resource state by cooling an appropriate interacting spin system [9, 10].

However, if we wish to adopt this viewpoint of the resource state for MBQC as the ground state of a quantum spin system, it would be too restrictive to confine ourselves to states in which the effect of measurements can be determined analytically from the tensor-network structure. A generic local Hamiltonian, or even an arbitrarily small generic local perturbation to a PEPS parent Hamiltonian, will not have such a property. Therefore, it is desirable to develop an understanding of MBQC in ground states of spin systems that does not rely on analytic control of the ground state. For this reason, there has been an interest in relating MBQC to forms of *quantum order* which, as parameters of the Hamiltonian are varied, can disappear only at a quantum phase transition [11–13].

In this paper, we use such a connection between MBQC and quantum order to give a precise characterization of the operation of MBQC in the ground states of a large class of perturbations to the 2-D cluster model. This allows us to give a rigorous proof that such perturbed ground states remain universal resources for MBQC provided that the perturbation is sufficiently small. Our proof relies in part on an extension of the the relationship we introduced in [13] between MBQC and *symmetry-protected topological (SPT) order* [14–16], a form of quantum order characterizing quantum systems which cannot be smoothly deformed into a product state while a certain symmetry is enforced. If the perturbation to the 2-D cluster model respects an appropriate symmetry, then the perturbed ground state will still possess non-trivial SPT order, and we show that this gives us sufficient information about the ground state to characterize the implications of the perturbation for MBQC. Our result therefore holds independently of any analytic solution for the perturbed ground state.

Our proof of universality is in the same spirit as Nielsen and Dawson's fault-tolerance proof for cluster state quantum computing [17]. There, it was shown that, whereas measurements on the cluster state simulate quantum circuits, measurements on a noisy cluster state simulate the same circuits, but with added noise. Here, our task is complicated by the highly correlated nature of the "errors" in the resource state that result from a change in the Hamiltonian. Nevertheless, we will show how to exploit the additional structure resulting from SPT order to establish an effective noise model for ground states of appropriate perturbed cluster models. Therefore, universal quantum computation can be achieved (for sufficiently small perturbations, corresponding to sufficiently weak noise in the effective circuit model) by choosing a measurement protocol which simulates a *fault-tolerant* quantum circuit. The universality is then a consequence of the *threshold theorem* [18] for fault-tolerant quantum computation with noisy quantum circuits.

Summary of results

Our ultimate goal in our paper is to prove the universality for a MBQC of a class of perturbations of the 2-D cluster state. However, in order to reach this goal, most of our attention is devoted to a further elucidation of the relationship between SPT order and MBQC. We first explore this relationship in one-dimensional systems, where we have previously shown that, in a class of quantum phases characterized by SPT order, the structure implied by SPT order leads to the perfect operation of the identity gate in MBQC [13]. By considering the 1-D cluster model, which lies in the simplest of the SPT phases considered in [13], we characterize the operation of non-trivial (i.e., not the identity) gates in the presence of a perturbation which respects the symmetry protecting this SPT phase. We obtain the following:

Theorem 1 (Effective noise model in one dimension)**.** *Consider a measurement protocol which in the exact 1-D cluster model would simulate a sequence of gates. In the perturbed resource state, the same measurement protocol simulates the same gate sequence, but with additional noise associated with each non-trivial gate. So long as the non-trivial gates are sufficiently separated from each other by identity gates, this effective noise has no correlations between different time steps, i.e. it is Markovian.*

The proof of Theorem 1 is divided into two stages. First, we establish Theorem 1 for ground states which are *pure finitely-correlated states* (pFCS), a special case of *matrix-product states* (MPS). For such states, both the manifestations of SPT order [15, 16], and the effect of measurements [3] can be understood straightforwardly in terms of the tensor-network structure. The ideas leading to Theorem 1 can be understood most directly in this context. Second, we prove Theorem 1 for arbitrary ground states within the SPT phase.

We then extend these ideas to the 2-D cluster model. We construct an appropriate symmetry group, such that the following result is satisfied for symmetry-respecting perturbations.

Theorem 2 (Effective noise model in two dimensions)**.** *Consider a measurement protocol which in the exact 2-D cluster model would simulate a sequence of gates. In the perturbed resource state, the same measurement protocol simulates the same gate sequence, but with additional noise associated with each gate. So long as the non-trivial gates are sufficiently separated from each other by identity gates, this effective noise has no correlations between different time steps, or between different gates taking place at the same time step, i.e., it is local and Markovian.*

Combined with the existing results on fault tolerance in the circuit model [18], Theorem 2 implies the main result of our paper:

Theorem 3. *For sufficiently small symmetry-respecting perturbations, the perturbed ground state remains a universal resource (using ideal measurements) for measurement-based quantum computation.*

Full details are found in Else, Bartlett, and Doherty, arXiv:1207.4815.

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